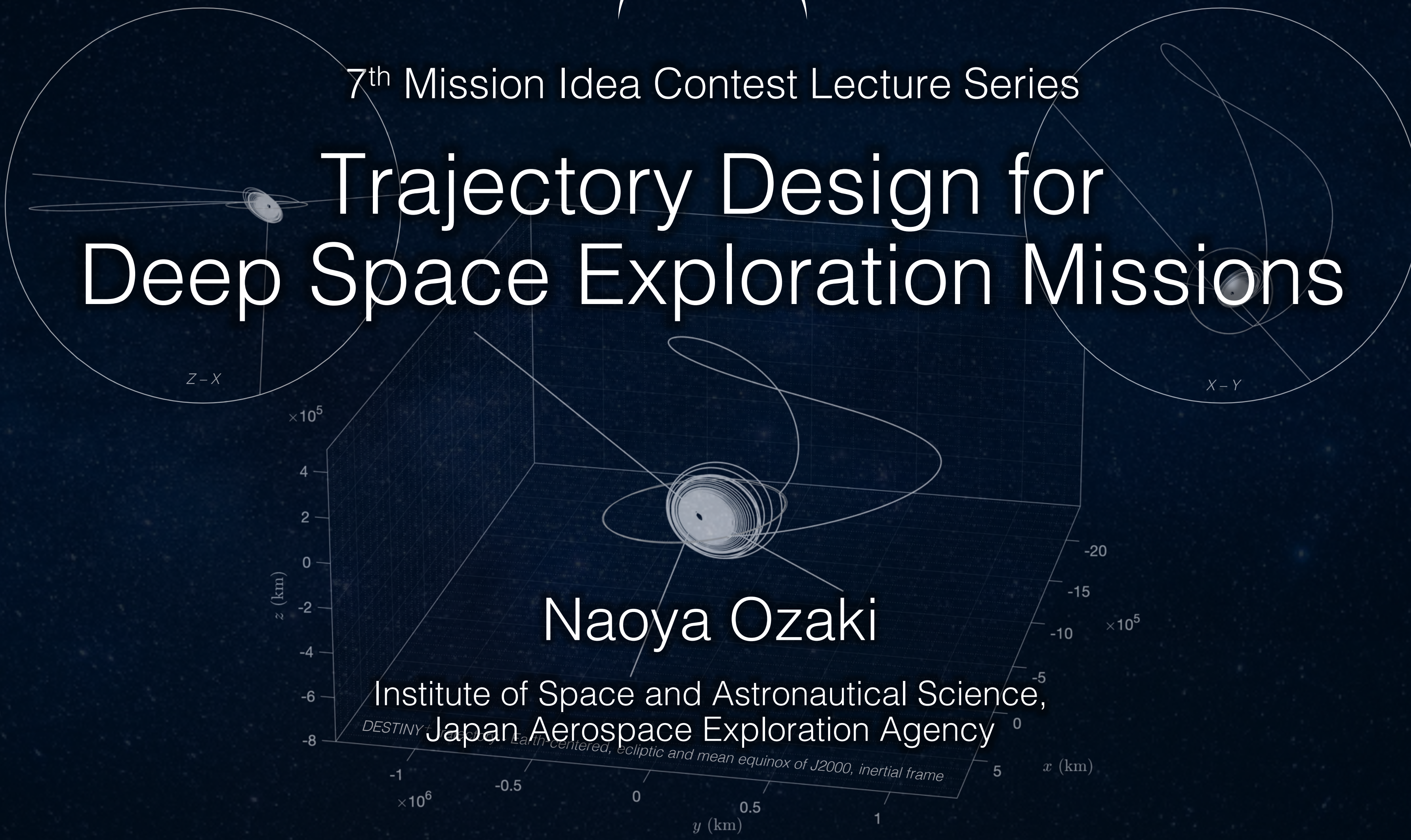
7th Mission Idea Contest Lecture Series

Trajectory Design for Deep Space Exploration Missions

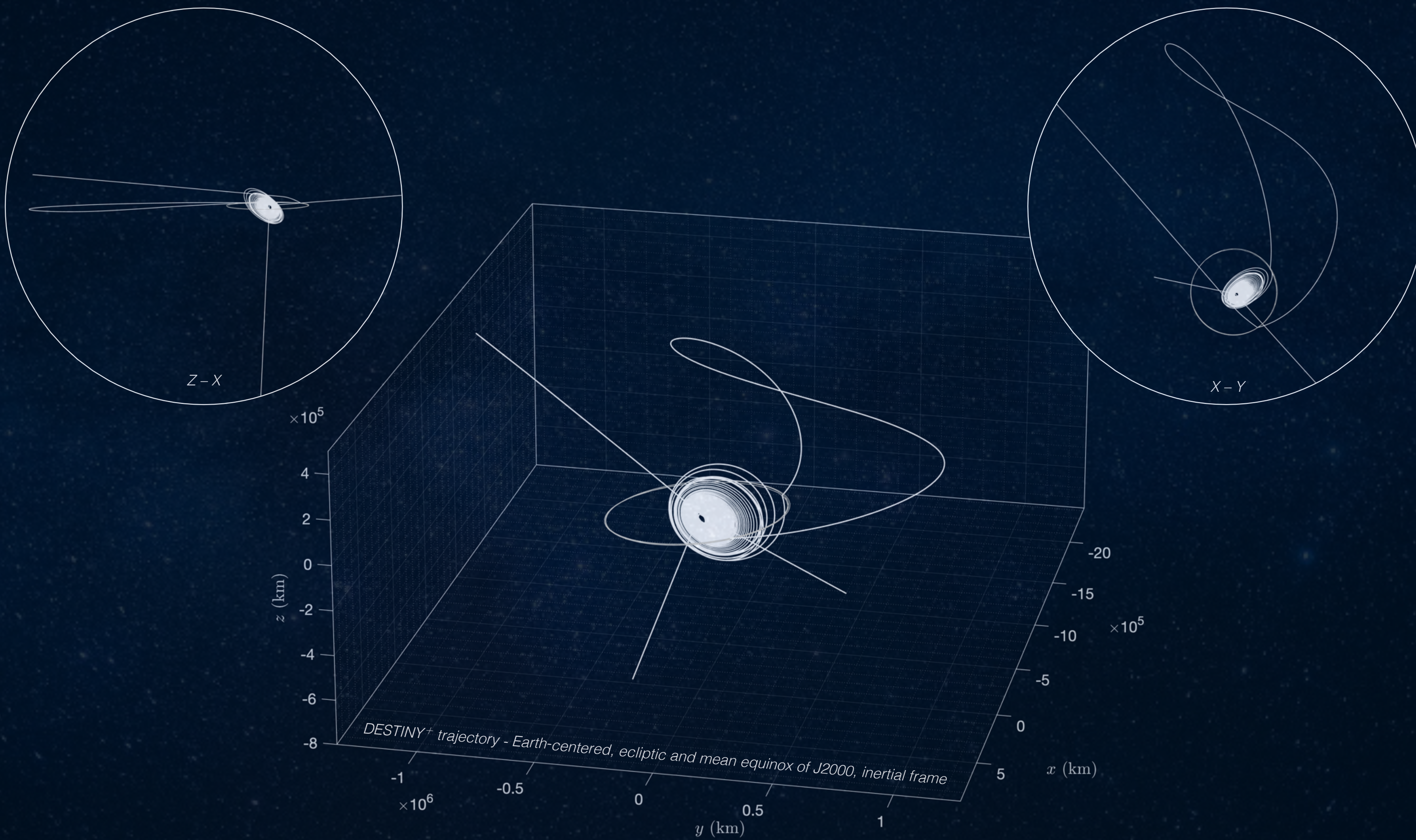
Naoya Ozaki

Institute of Space and Astronautical Science,
Japan Aerospace Exploration Agency



DESTINY+ Trajectory

Utilizing electric propulsion, lunar swing-bys, and solar tidal force



1. Introduction

Self Introduction

Naoya Ozaki

Assistant Professor (Tenure-track) at JAXA/ISAS

Space Mission Designer, Astrodynamicist

Biography

2010-2015: B.S. & M.S. University of Tokyo
(Under Prof. Shinichi Nakasuka and Prof. Ryu Funase)

2015-2018: Ph.D. University of Tokyo, JSPS Research Fellow(DC1)

2016: ESA/ESOC, Mission Analysis Section, Trainee

2017: NASA/JPL, Mission Design and Navigation Section,
Outer Planet Mission Design Group, Research Intern

2018 April-: JAXA/ISAS, JSPS Research Fellow (PD)

2019 March-: JAXA/ISAS Assistant Professor (Tenure-track)

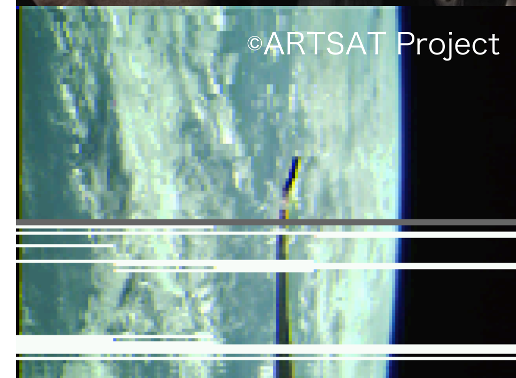
Projects

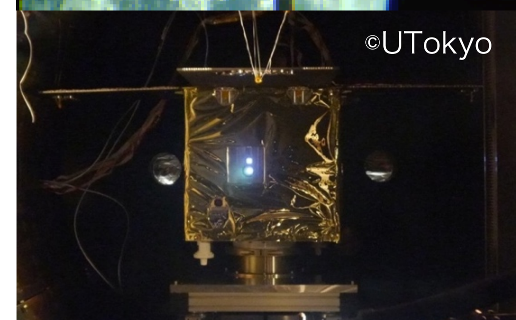
2013-2016: PROCYON (launched in 2014 with Hayabusa-2)

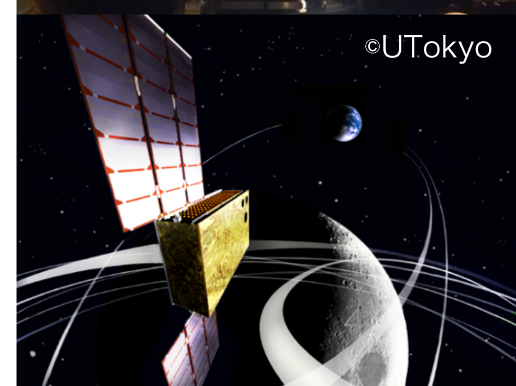
2015- : EQUULEUS

2018- : MMX

2019- : DESTINY$^+$, Comet Interceptor



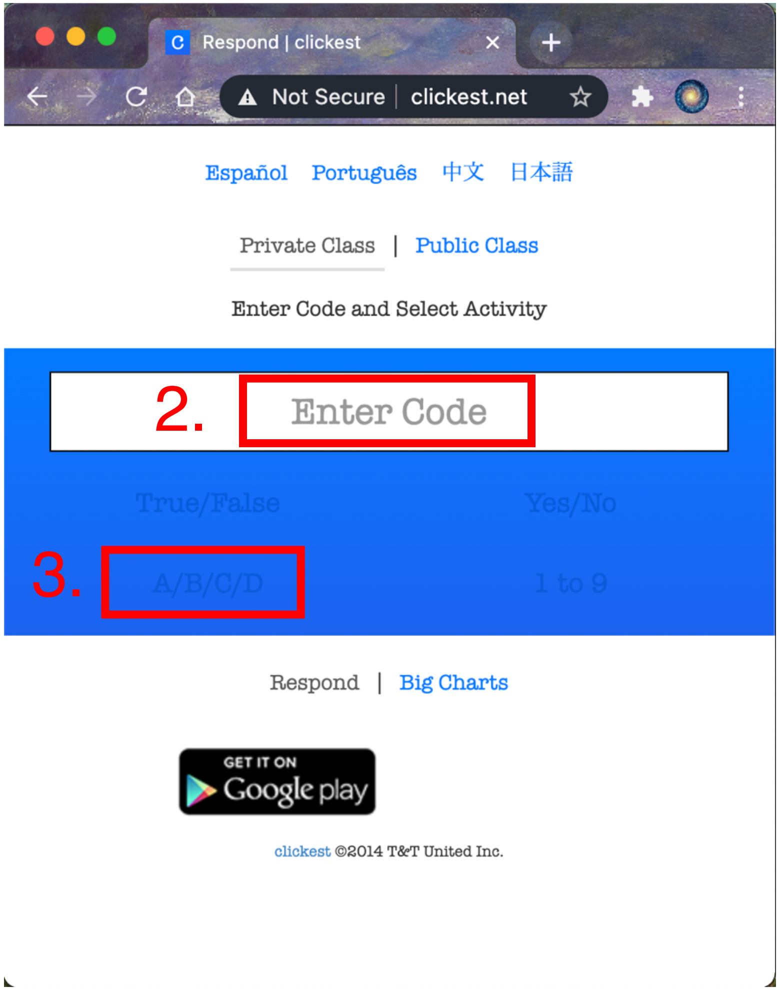




Clickest App

Please access Clickest website
http://www.clickest.net/

1. Click「Respond >>」
2. Enter the code「ayitqs」
3. Select「A/B/C/D」
4. Answer questions



Test Clickest App

Question：What is the official name Japanese space agency?

A

National Space Development Agency of Japan

B

Japan Aerospace Exploration Agency

C

Japanese Space Agency

D

Team Rocket

clickest

Code: ayitqs

Test Clickest App

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Japanese Space Agency

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Team Rocket

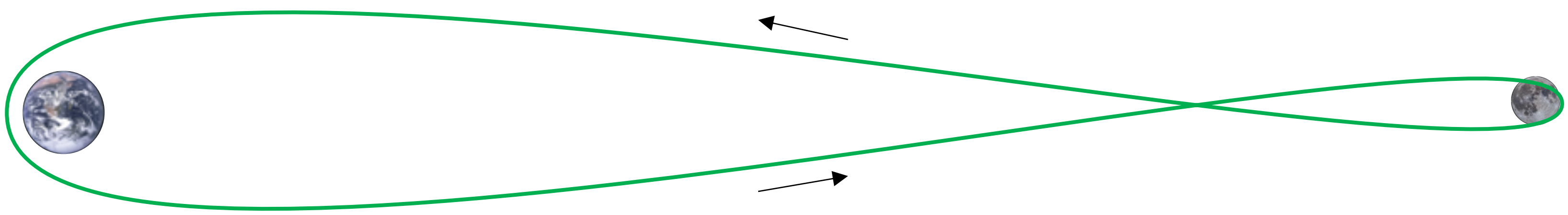
Goal of This Lecture

- ☑ To be able to explain the role of trajectory design in deep space exploration missions.
- ☑ To be able to explain what the Hohmann transfer orbit, patched conics, and swing-by.
- ☑ To understand the brief overview of the advanced techniques of astrodynamics

2. Fundamentals of Astrodynamics

What is Trajectory Design?

＝to determine the route the spacecraft will take



Roundtrip trajectory toward the Moon

What is Trajectory Design?

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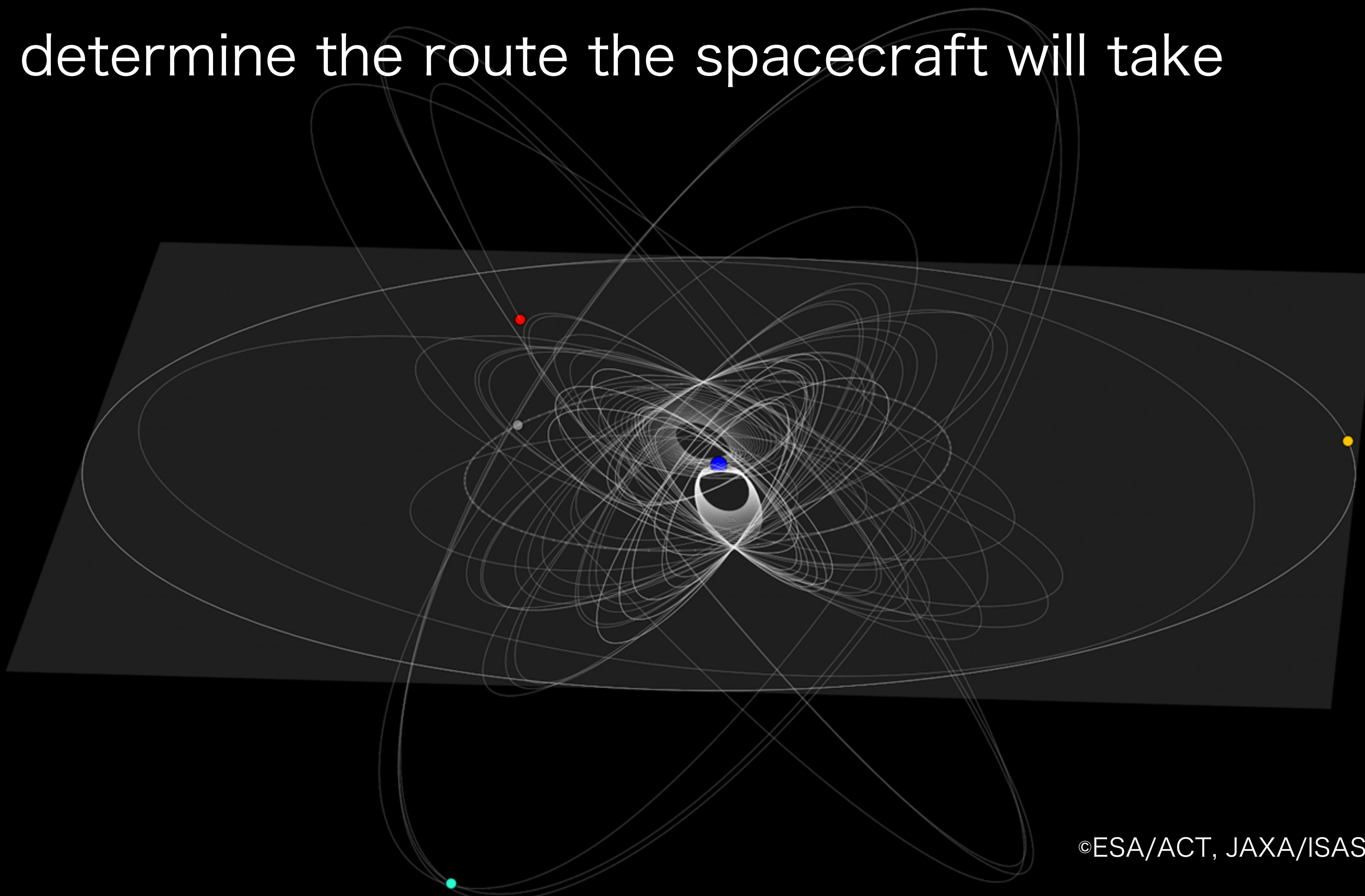
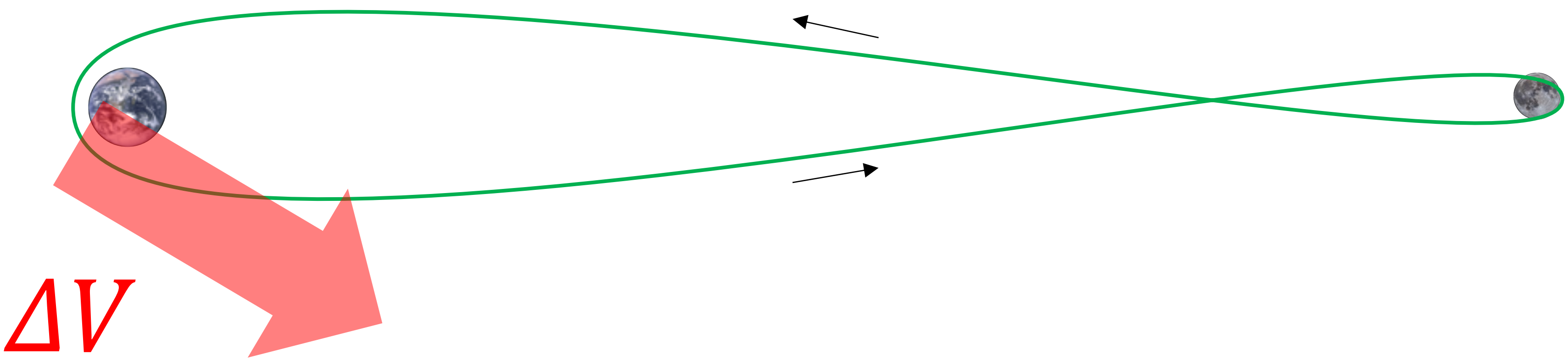


Trajectory of GTOC8 (8^{th} Global Trajectory Optimization Competition)

What is Trajectory Design?

＝to determine the route the spacecraft will take



Roundtrip trajectory toward the Moon

How do we gain ΔV??

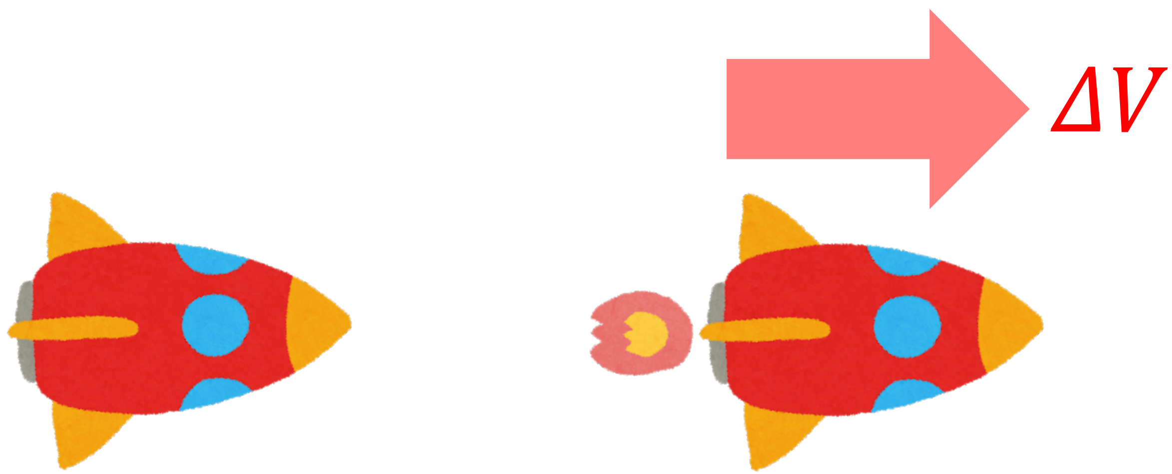
How do we gain ΔV??

The commonly used method is ***rocket propulsion***.
(There are some other tricks)

How does a rocket work?

= Conservation of Momentum

If you throw a "large” mass backwards "fast",
you will gain a large ΔV from the reaction.



How large mass can a rocket carry?

clickest

Code: ayitqs

Rocket Equation

For an initial mass m_i, a final mass m_f, and a specific impulse I_{sp}, **which rocket equation is correct??**

Remember Prof. Funase and Prof. Koizumi's Lecture!!!

A

$$m_f = m_i \ln\left(\frac{\Delta V}{g_0 I_{sp}}\right)$$

B

$$m_f = m_i \exp\left(\frac{\Delta V}{g_0 I_{sp}}\right)$$

C

$$m_f = m_i \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right)$$

D

$$m_f = m_i \ln\left(-\frac{\Delta V}{g_0 I_{sp}}\right)$$

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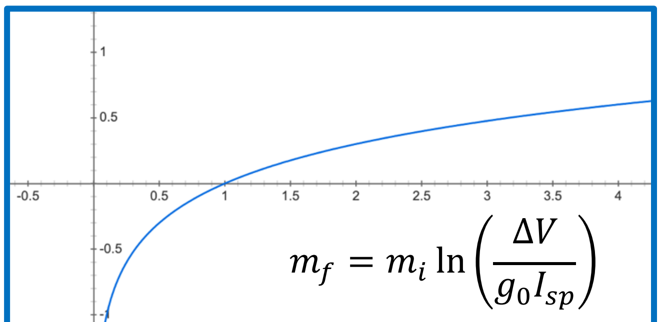
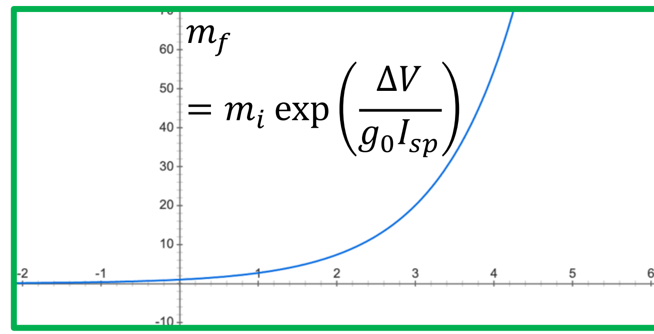
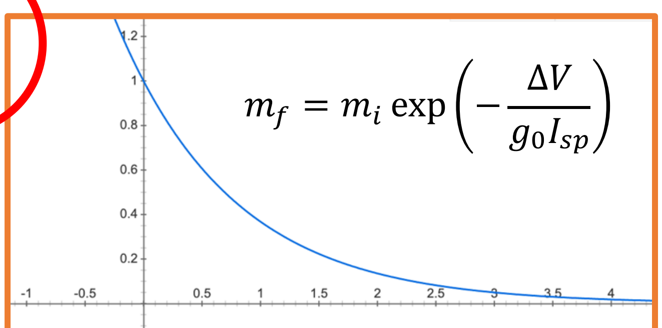


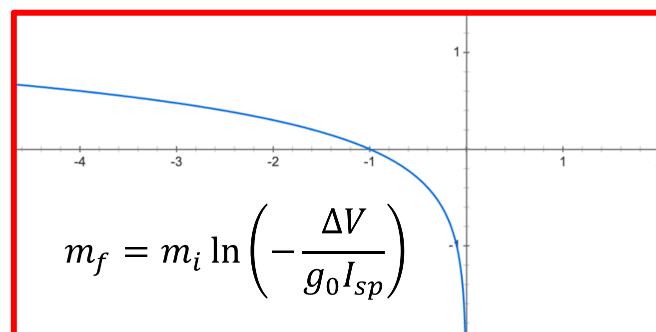
B



C



D



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Rocket Equation

For an initial mass m_i, a final mass m_f, and a specific impulse I_{sp}, **which rocket equation is correct??**

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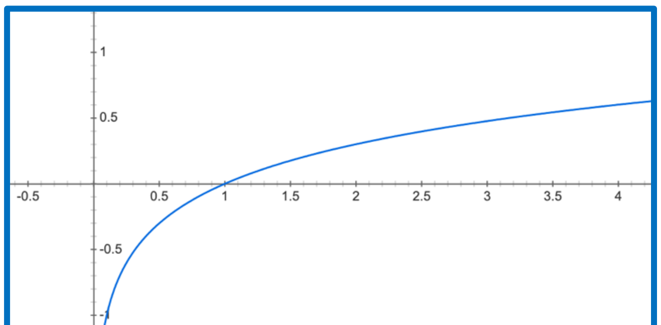
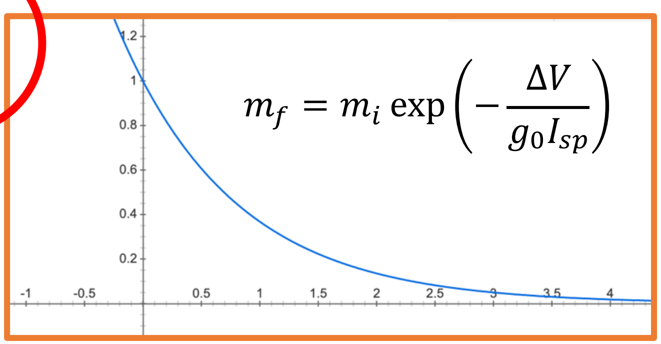


C



For a given initial mass m_i, larger m_f is better.

How can we get larger m_f??

- Make I_{sp} larger

  (research subject of space propulsion specialist)

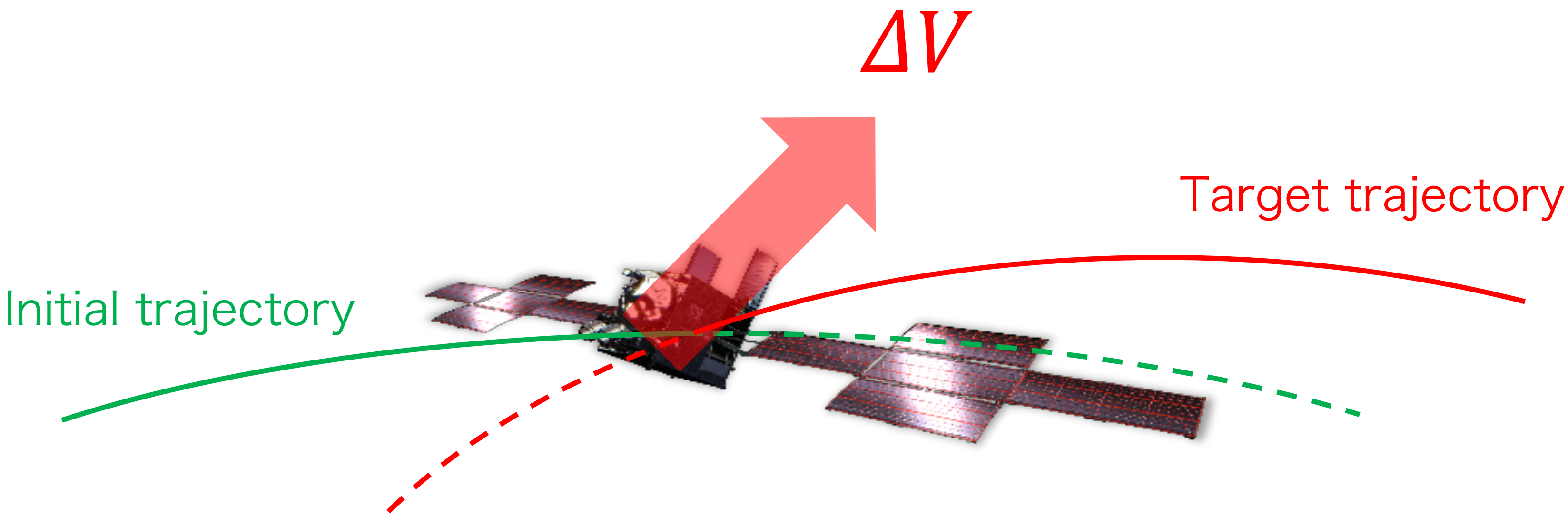
- Make ΔV smaller

  (research subject of mission designer)

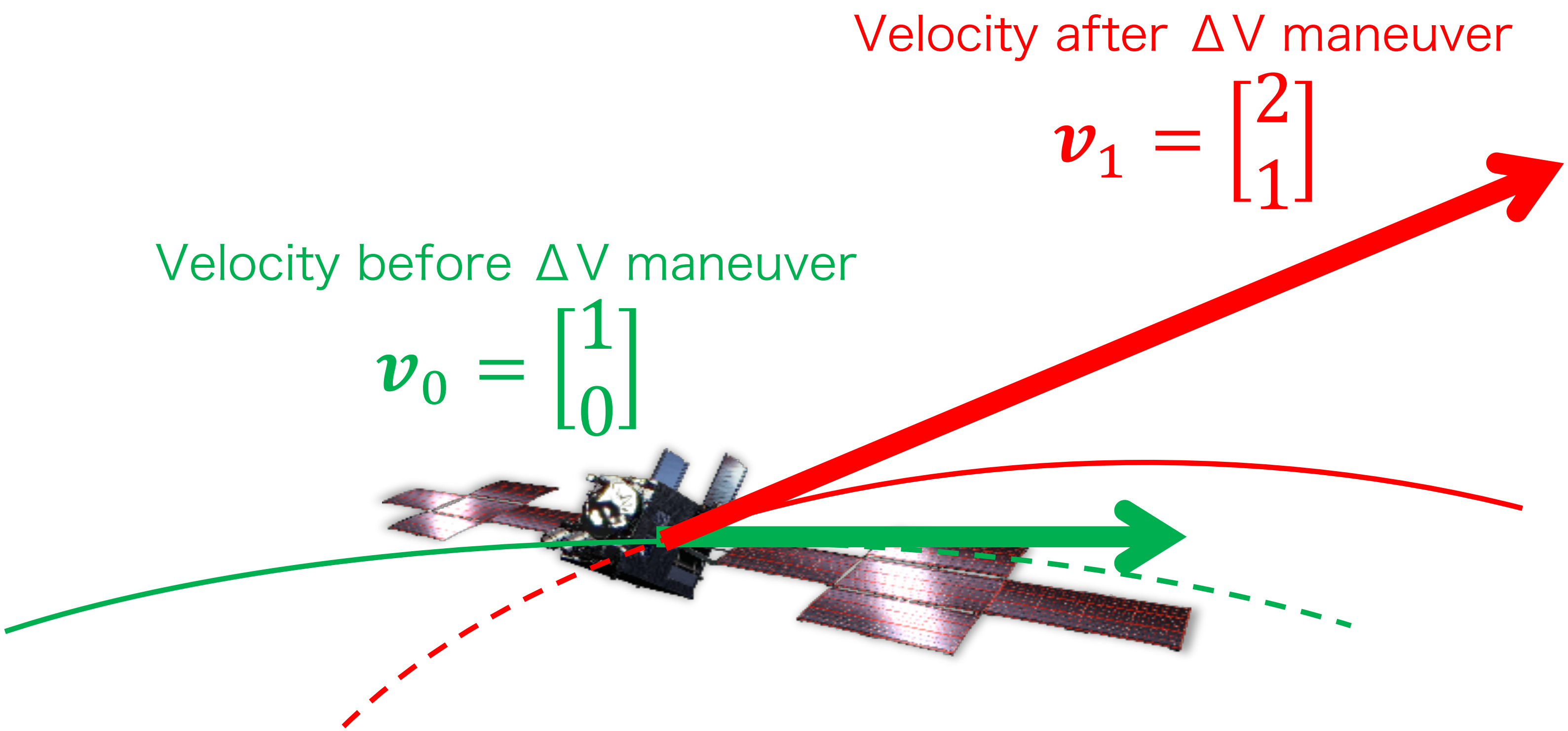
So, how do we calculate ΔV??

＝Just calculate
the difference in orbital velocities

How to Calculate ΔV

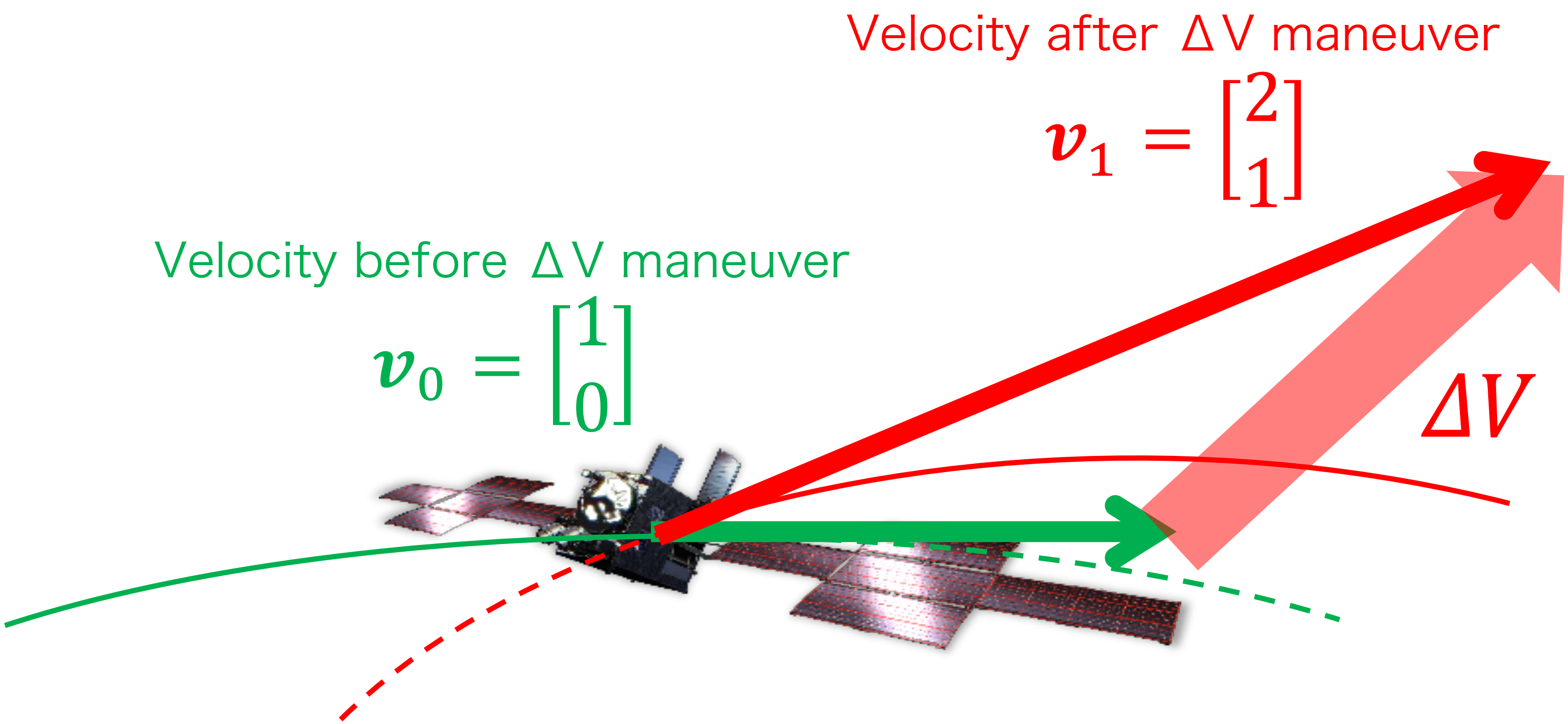


How to Calculate ΔV



…What is the magnitude of the velocity difference?

How to Calculate ΔV



$$\Delta V = \|\boldsymbol{v}_1 - \boldsymbol{v}_0\| = \sqrt{(2-1)^2 + (1-0)^2} = \sqrt{2}$$

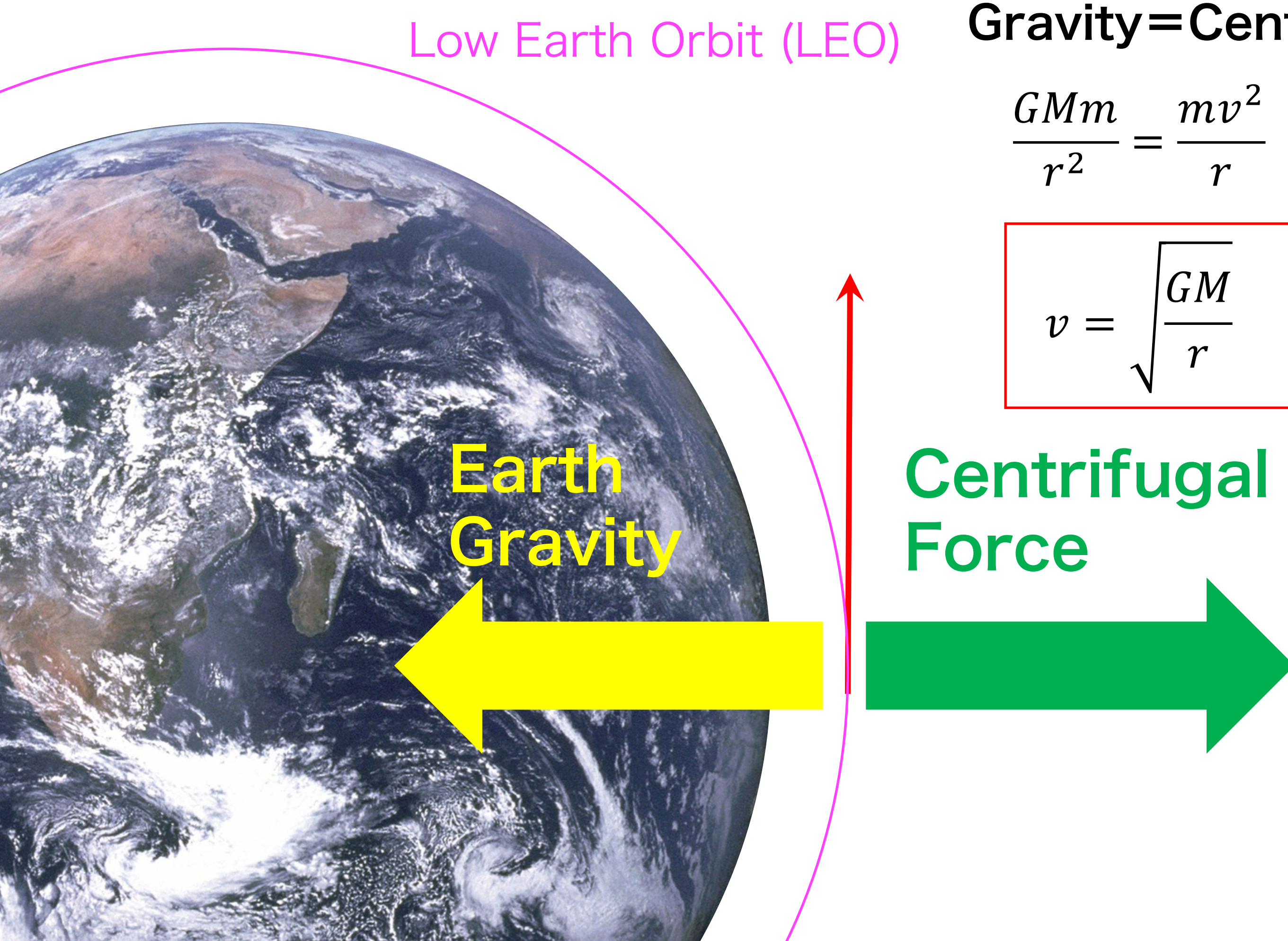
Easy, huh??

Then, how do you calculate the orbital velocity vector?

That is the essence of trajectory design!

Orbital Velocity: Low Earth Orbit

How much velocity do we need to orbit around the Earth?



Gravity=Centrifugal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

G: constant of gravitation
M: Earth mass
m: spacecraft mass
r: orbital radius
v: orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

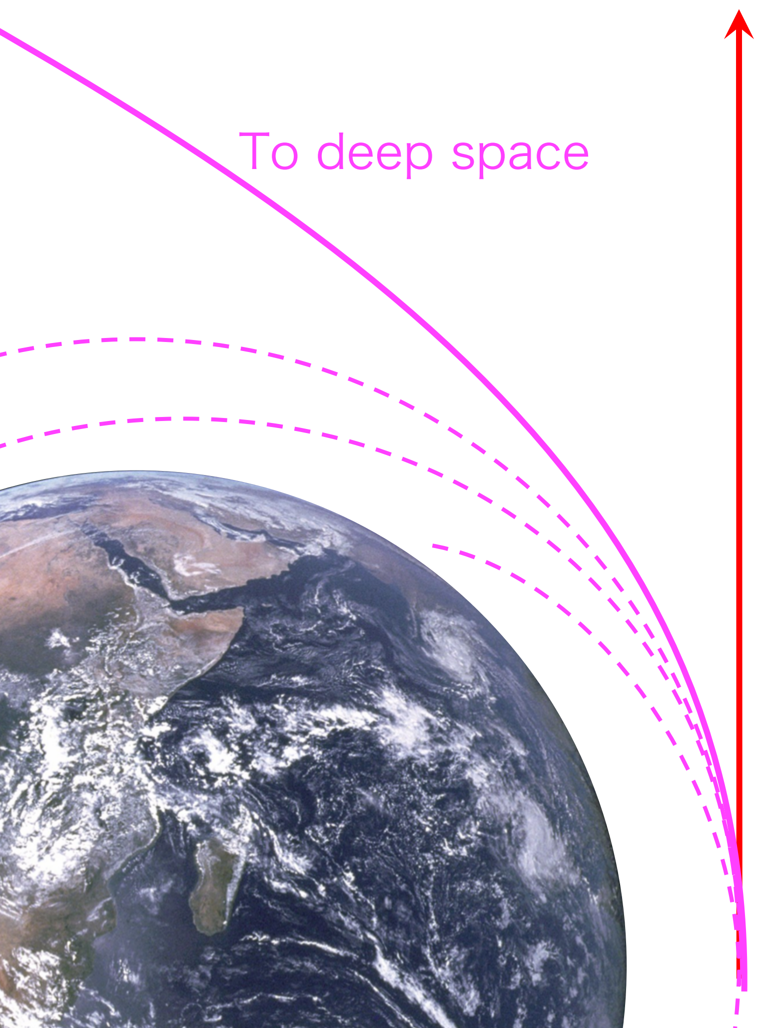
Substituting $r = 6378\ \mathrm{km}$ yields

$$v = 7.9\mathrm{km/s}$$

(First cosmic velocity)

Orbital Velocity: To Deep Space

How much velocity do we need to escape from the Earth?



Kinetic energy > Potential energy

$$\frac{1}{2}mv^2 > \frac{GMm}{r}$$

$$v > \sqrt{\frac{2GM}{r}}$$

Substituting $r = 6378\,\mathrm{km}$ yields

$$v > 11.2\mathrm{km/s}$$

(Second cosmic velocity
= Escape velocity)

ΔV from LEO to deep space

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Code: ayitqs

Calculate ΔV from Low Earth Orbit (LEO) to deep space.
(If you finish answering, calculate m_f/m_i where $I_{sp} = 280s$)
*Typical Isp of solid rocket motor

A

$\Delta V = 3.3\mathrm{m/s}$

B

$\Delta V = 33\mathrm{m/s}$

C

$\Delta V = 330\mathrm{m/s}$

D

$\Delta V = 3.3\mathrm{km/s}$

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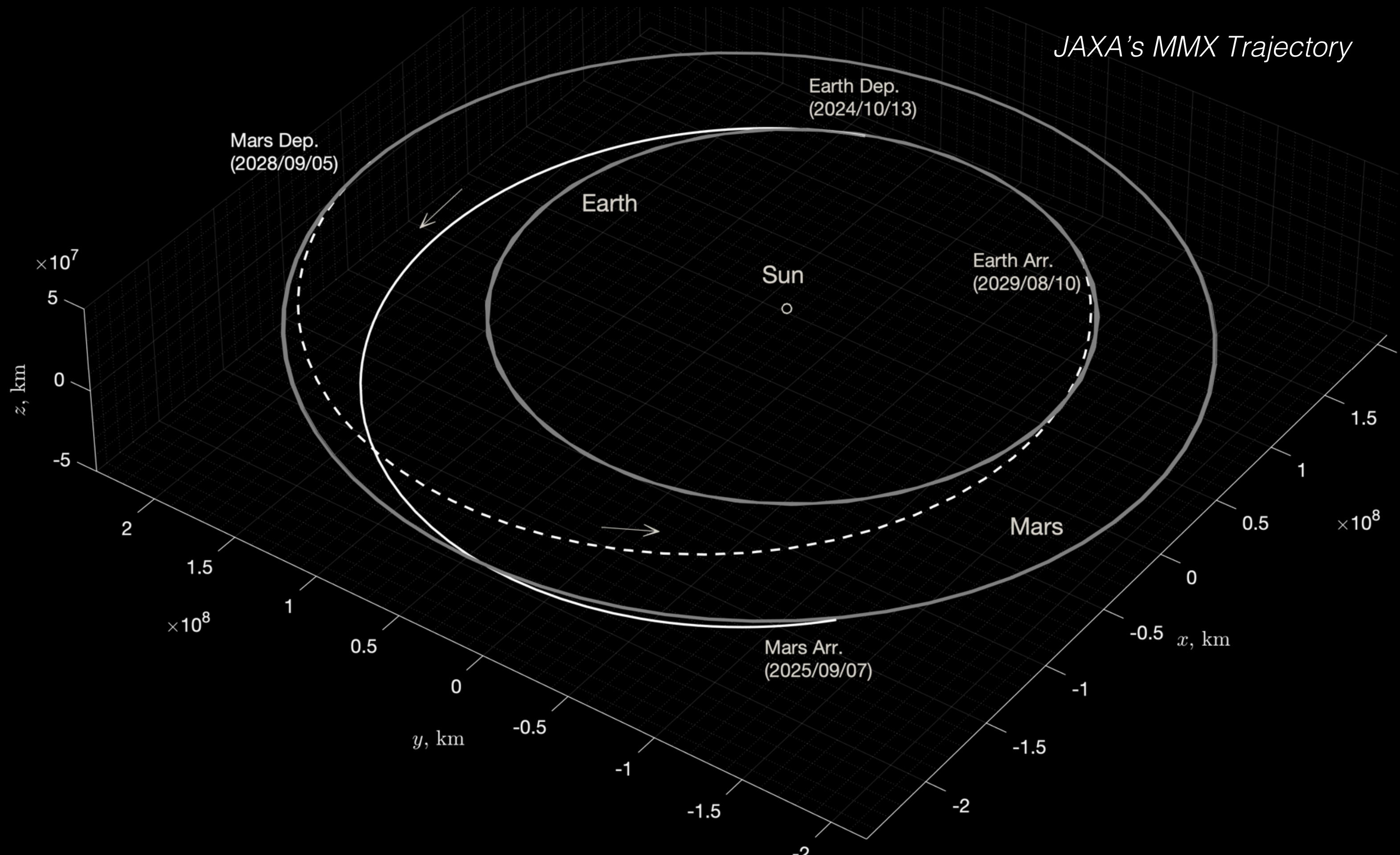
$\Delta V = 3.3\mathrm{km/s}$

$$\frac{m_f}{m_i} = \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right) = \exp\left(-\frac{3300}{9.8\times 280}\right) = 0.3004$$

*70% of the initial mass m_i is the propellant mass!!



Now let's go to Mars.

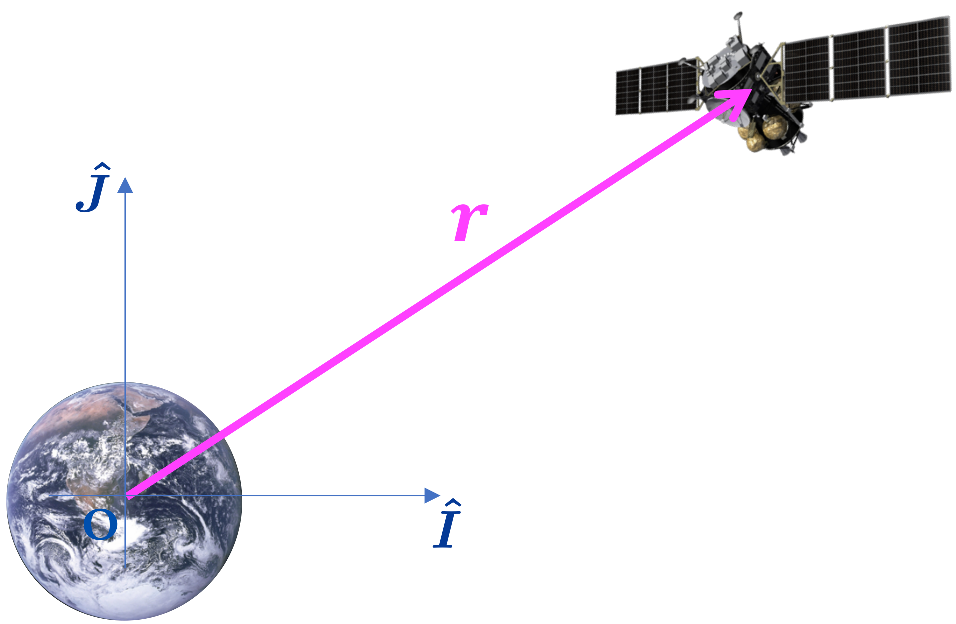
Equation of Motion of Two-body Problem

Newton's law of universal gravitation
(vector form)

$$m\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GMm}{r^2}\frac{\boldsymbol{r}}{r}$$

Hense,

$$\boxed{\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM}{r^3}\boldsymbol{r}}$$



Sir Isaac Newton
(1643 - 1727)

Conserved Quantity of Two-body Problem

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM}{r^3}\boldsymbol{r}$$

For analytical discussion of a dynamical system, **conserved quantities** play important roles!!

- Conservation of Angular momentum
- Conservation of Energy

Conservation of Angular Momentum

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM}{r^3}\boldsymbol{r}$$

Rewrite the equation with $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$.

$$\frac{d\boldsymbol{v}}{dt} = -\frac{GM}{r^3}\boldsymbol{r}$$

Fast forward

Conservation of Angular Momentum

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Rewrite the equation with $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$.

$$\frac{d\boldsymbol{v}}{dt} = -\frac{GM}{r^3}\boldsymbol{r}$$

Take the cross product of $\boldsymbol{r}$ from the left for both sides.

$$\boldsymbol{r}\times\frac{d\boldsymbol{v}}{dt} = -\boldsymbol{r}\times\frac{GM}{r^3}\boldsymbol{r} = -\frac{GM}{r^3}\boldsymbol{r}\times\boldsymbol{r}$$

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Using the relation

$$\frac{d}{dt}(\boldsymbol{r}\times\boldsymbol{v}) = \frac{d\boldsymbol{r}}{dt}\times\boldsymbol{v} + \boldsymbol{r}\times\frac{d\boldsymbol{v}}{dt} = \boldsymbol{v}\times\boldsymbol{v} + \boldsymbol{r}\times\frac{d\boldsymbol{v}}{dt} = \boldsymbol{r}\times\frac{d\boldsymbol{v}}{dt}$$

Fast forward ▶▶

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yields

$$\frac{d}{dt}(\boldsymbol{r}\times\boldsymbol{v}) = \boldsymbol{0}$$

Fast forward ▶▶

Conservation of Angular Momentum

$$\frac{d}{dt}(\boldsymbol{r}\times\boldsymbol{v}) = \mathbf{0}$$

Here, we define the angular momentum

$$\boldsymbol{h} \coloneqq \boldsymbol{r}\times\boldsymbol{v}$$

Finally, we obtain the conservation of the angular momentum

$$\boxed{\boldsymbol{h} = \boldsymbol{r}\times\boldsymbol{v} = const}$$

From the equation of motion, the conservation of angular momentum was derived!
(From Newton's law to Kepler's law)

Fast forward

Conservation of Energy

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM}{r^3}\boldsymbol{r}$$

Rewrite the equation with $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$.

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Take the dot product of $\boldsymbol{v}$ for both sides.

$$\boldsymbol{v} \cdot \frac{d\boldsymbol{v}}{dt} = -\frac{GM}{r^3}\boldsymbol{r} \cdot \frac{d\boldsymbol{r}}{dt}$$

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Using

$$\frac{d}{dt}\frac{v^2}{2} = \frac{d}{dt}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{v}}{2}\right) = \boldsymbol{v} \cdot \frac{d\boldsymbol{v}}{dt} \text{ and } \frac{d}{dt}\frac{r^2}{2} = \frac{d}{dt}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{r}}{2}\right) = \boldsymbol{r} \cdot \frac{d\boldsymbol{r}}{dt}$$

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yields

$$\frac{d}{dt}\frac{v^2}{2} = -\frac{GM}{r^3}\frac{d}{dt}\frac{r^2}{2}$$

Fast forward

Conservation of Energy

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yields

$$\frac{d}{dt}\frac{v^2}{2} = -\frac{GM}{r^3}\frac{d}{dt}\frac{r^2}{2}$$

$$\frac{d}{dt}v^2 = -GM\,(r^2)^{-\frac{3}{2}}\frac{d}{dt}r^2$$

$$\frac{d}{dt}v^2 = -GM\,\frac{d}{dt}\left\{-2(r^2)^{-\frac{1}{2}}\right\}$$

Conservation of Energy

$$\frac{d}{dt}v^2 = -GM\frac{d}{dt}\left\{-2\left(r^2\right)^{-\frac{1}{2}}\right\}$$

$$\frac{d}{dt}\left\{v^2 - \frac{2GM}{r}\right\} = 0$$

Here, we define the energy

$$E := \frac{1}{2}v^2 - \frac{GM}{r}$$

Finally, we obtain the conservation of the energy

$$\boxed{E = \frac{1}{2}v^2 - \frac{GM}{r} = const}$$

From the equation of motion, the conservation of energy was derived!

Conserved Quantity of Two-body Problem

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- Conservation of Angular momentum

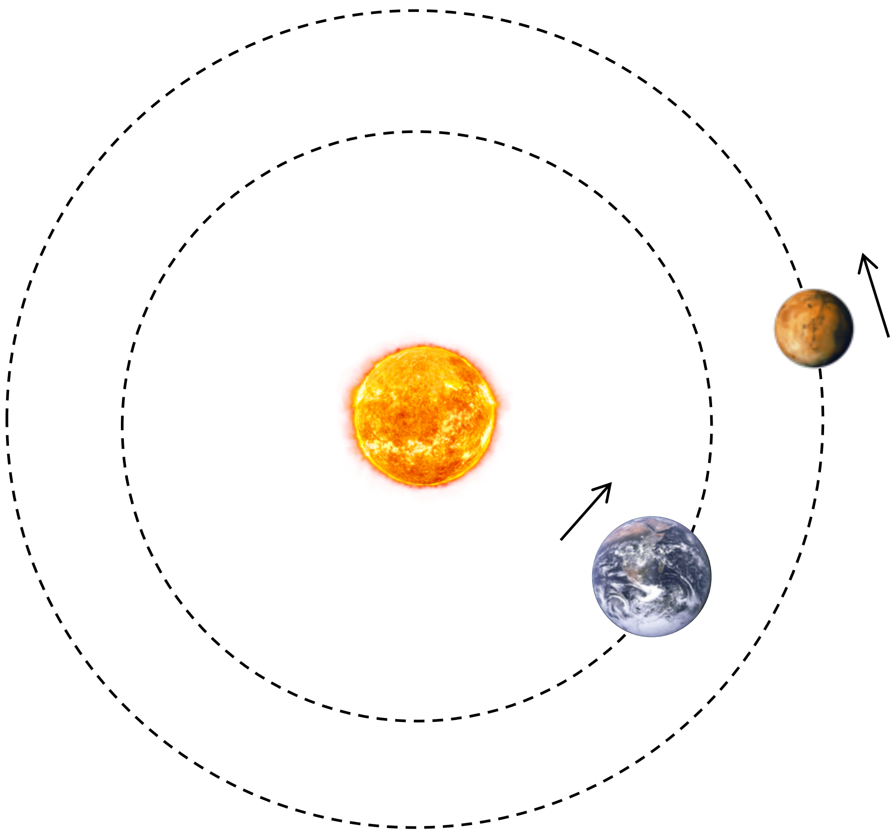
$$\boldsymbol{r}\times\boldsymbol{v} = const$$

- Conservation of Energy

$$\frac{1}{2}v^2 - \frac{GM}{r} = const$$

Hohmann Transfer Orbit

What is the transfer orbit that consumes the minimum ΔV fly to Mars?



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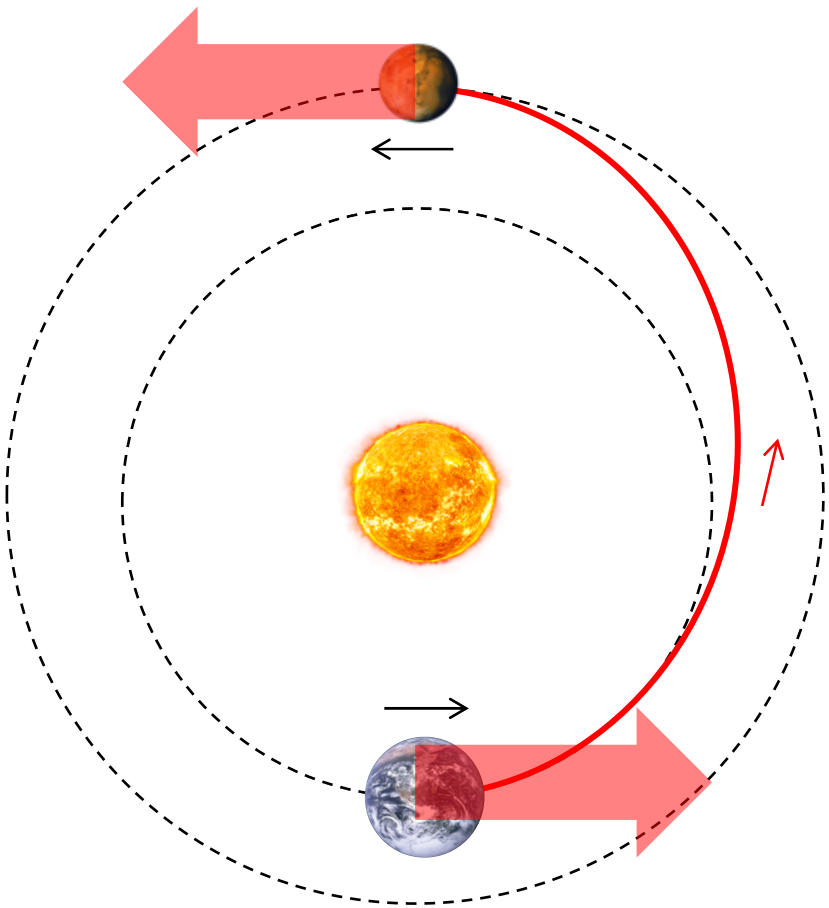

The minimum ΔV transfer is achieved when the spacecraft can meet planets in opposite (180 degree) positions!

Hohmann Transfer Orbit

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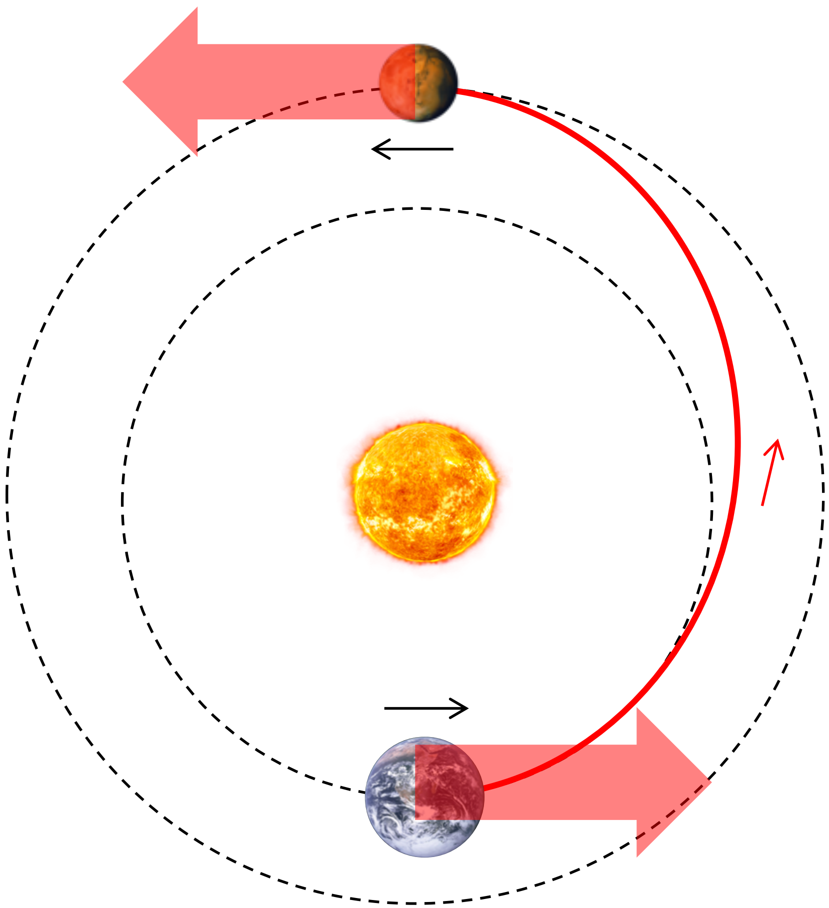

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Hohmann Transfer Orbit

The first step to obtain the Hohmann transfer ΔV is calculating the perihelion and aphelion velocities.

Exercise: Perihelion and Aphelion Velocities of An Elliptical Orbit

Given the perihelion radius r_p and aphelion radius r_a, find the perihelion velocity v_p and aphelion velocity v_a from the laws of conservation of energy and angular momentum.

Here, assume that the masses of the Earth, Mars, and the spacecraft are negligible.

Hint:

Energy conservation :

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}v_a^2 - \frac{GM}{r_a}$$

Angular momentum conservation:

$$r_p v_p = r_a v_a$$

Exercise: Perihelion and Aphelion Velocities of An Elliptical Orbit

Energy conservation

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}v_a^2 - \frac{GM}{r_a}$$

Angular momentum conservation

$$r_p v_p = r_a v_a$$

Using these conservation laws

$$\frac{1}{2}v_p^2 - \frac{1}{2}\left(\frac{r_p}{r_a}\right)^2 v_p^2 = \frac{GM}{r_p} - \frac{GM}{r_a}$$

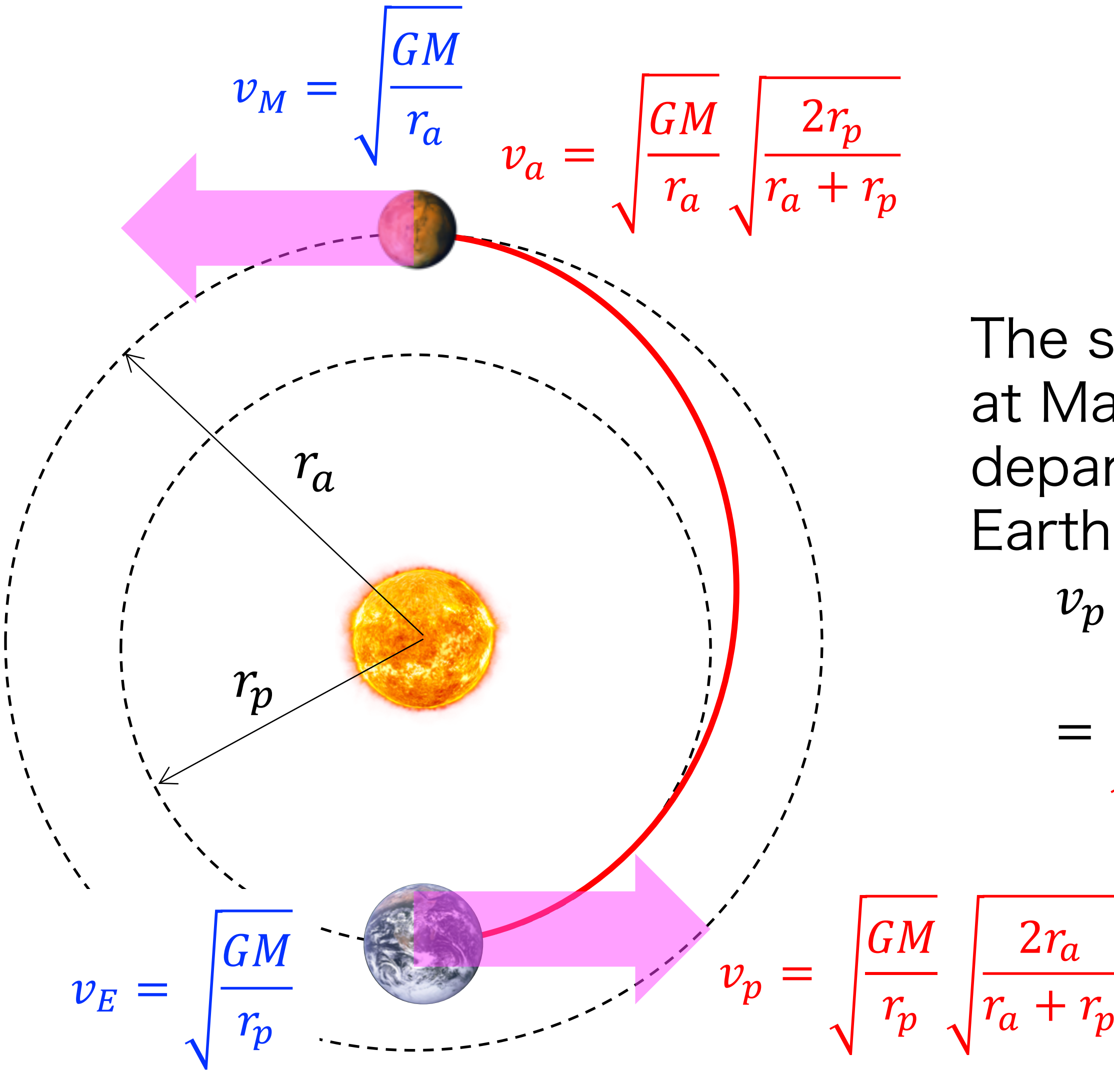
$$v_p = \sqrt{\frac{GM(r_a - r_p)}{r_p r_a}\frac{2r_a^2}{r_a^2 - r_p^2}}$$

$$v_p = \sqrt{\frac{GM}{r_p}}\sqrt{\frac{2r_a}{r_a + r_p}}$$

In the same way,

$$v_a = \sqrt{\frac{GM}{r_a}}\sqrt{\frac{2r_p}{r_a + r_p}}$$

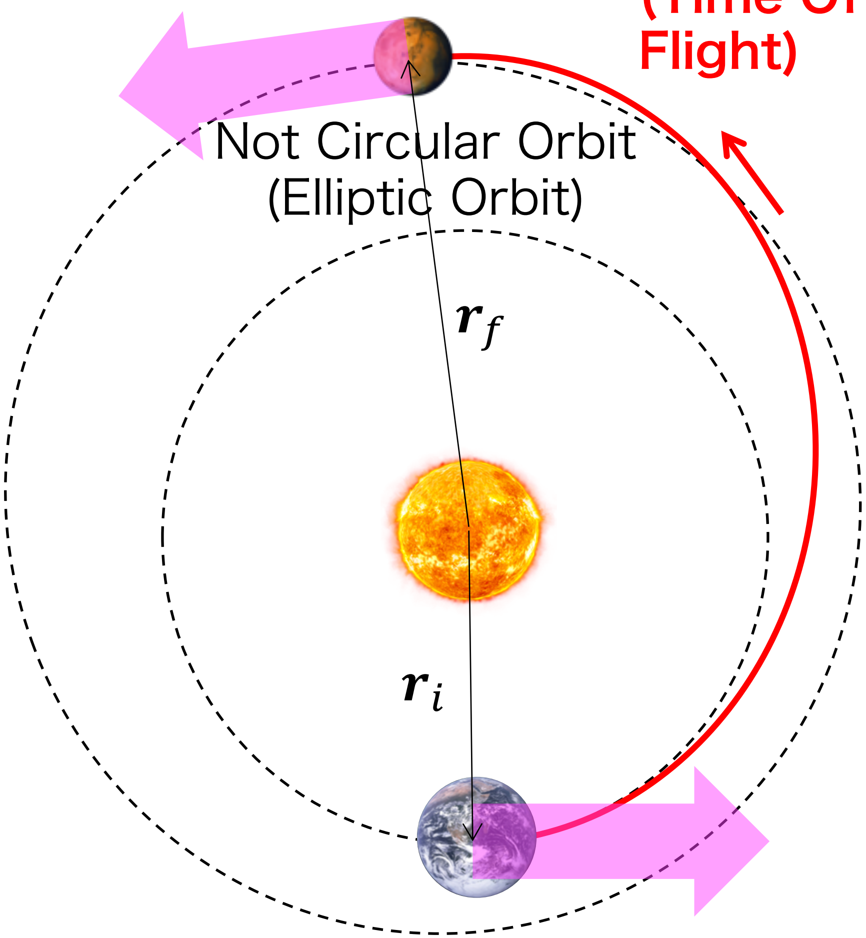
Hohmann Transfer Orbit



The spacecraft can arrive at Mars when the departure velocity from the Earth is

$$v_p - v_E = \sqrt{\frac{GM}{r_p}}\sqrt{\frac{2r_a}{r_a + r_p}} - \sqrt{\frac{GM}{r_p}}$$

Related Topic: Lambert's Problem



The Hohmann transfer orbit assumes the circular co-planar orbits, but both the Earth and Mars move elliptic orbit.

Lambert's Problem:

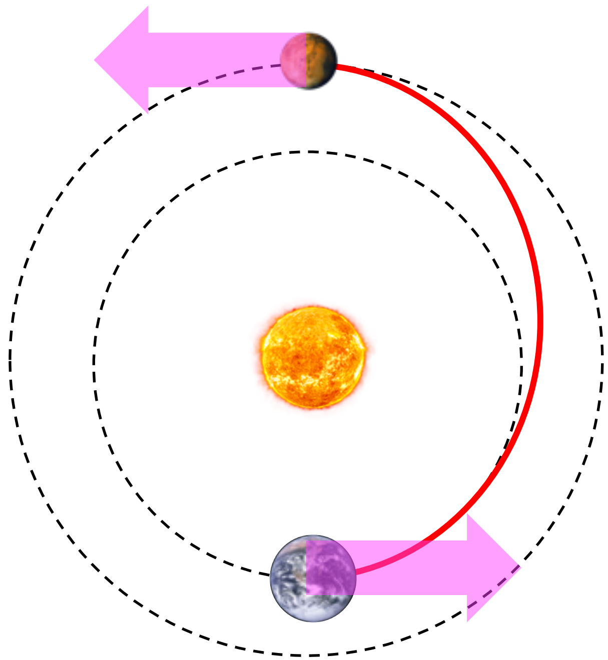
$$\boldsymbol{r}_i, \boldsymbol{r}_f, \mathrm{TOF} \mapsto \boldsymbol{v}_i, \boldsymbol{v}_f$$

Example of Lambert's problem solver: PyKep (Python)

See details:

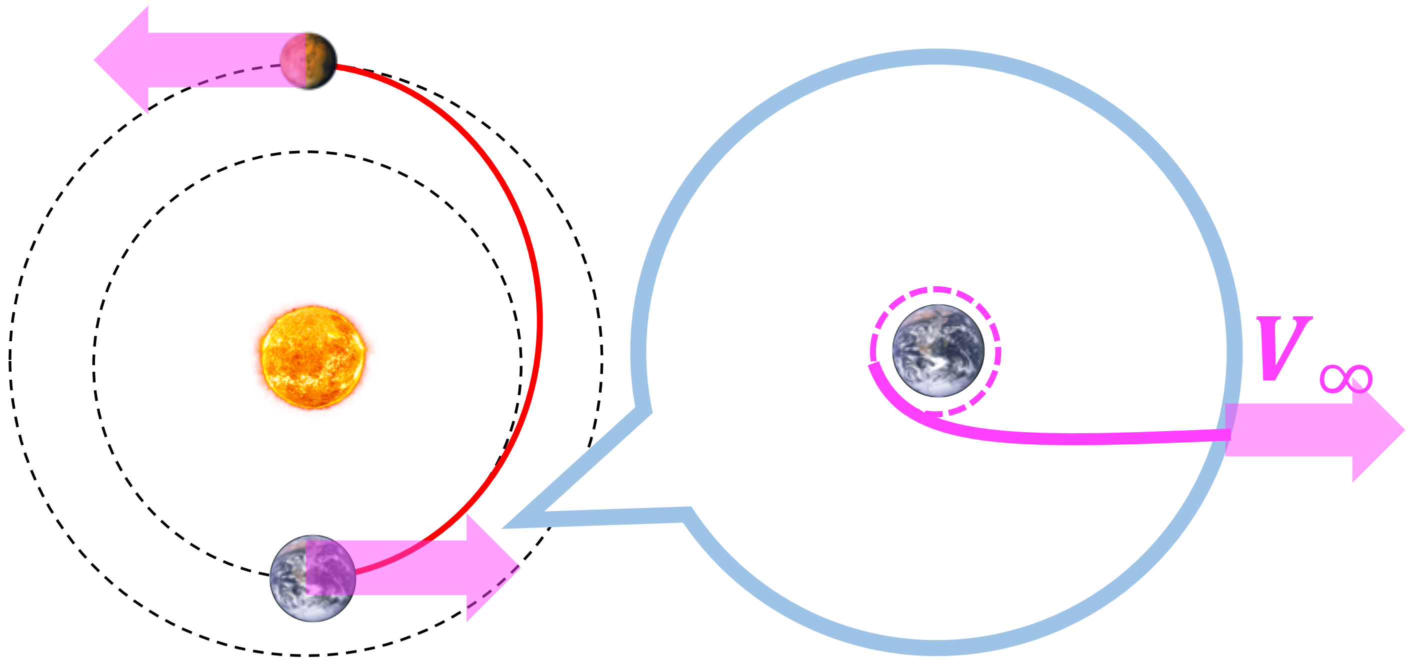
- R. H. Battin, An Introduction to the Mathematics and Methods of Astrodynamics, Section 7.
- R. H. Gooding, A Procedure for Solution of Lambert's Orbital Boundary-Value Problem, Celestial Mechanics and Dynamical Astronomy, Vol.48, pp.145-165, 1990.
- D. Izzo, Revisiting Lambert's Problem, Celestial Mechanics and Dynamical Astronomy, Vol.121, pp.1-15, 2015.

In Reality…
(Considering the Gravity of the Earth)



In the vicinity of the Earth (inside the sphere of influence), the influence of the Earth's gravity becomes dominant.

In Reality…
(Considering the Gravity of the Earth)



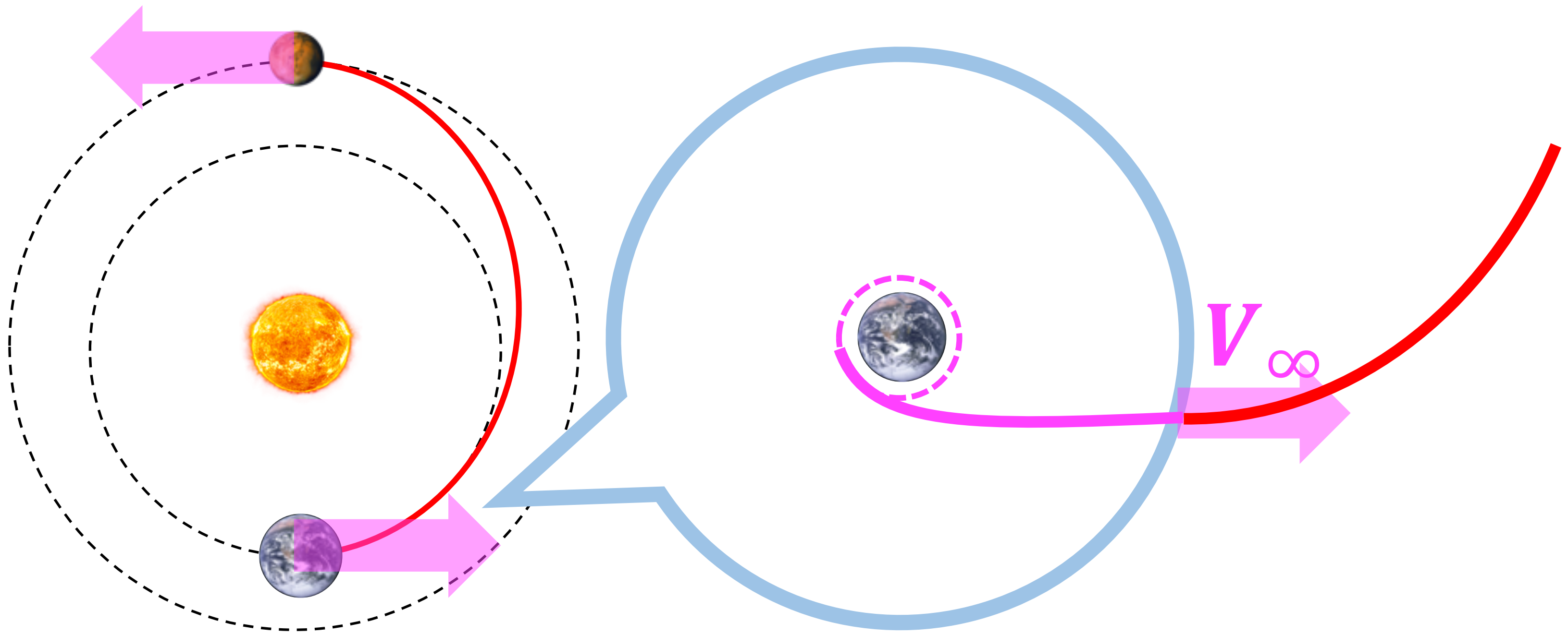
If the velocity at a point sufficiently far from the Earth becomes

$$(V_\infty =)\ v_p - v_E = \sqrt{\frac{GM}{r_p}}\sqrt{\frac{2r_a}{r_a + r_p}} - \sqrt{\frac{GM}{r_p}}$$

(= Velocity difference of the Hohmann Transfer orbit)

then the spacecraft can reach Mars.

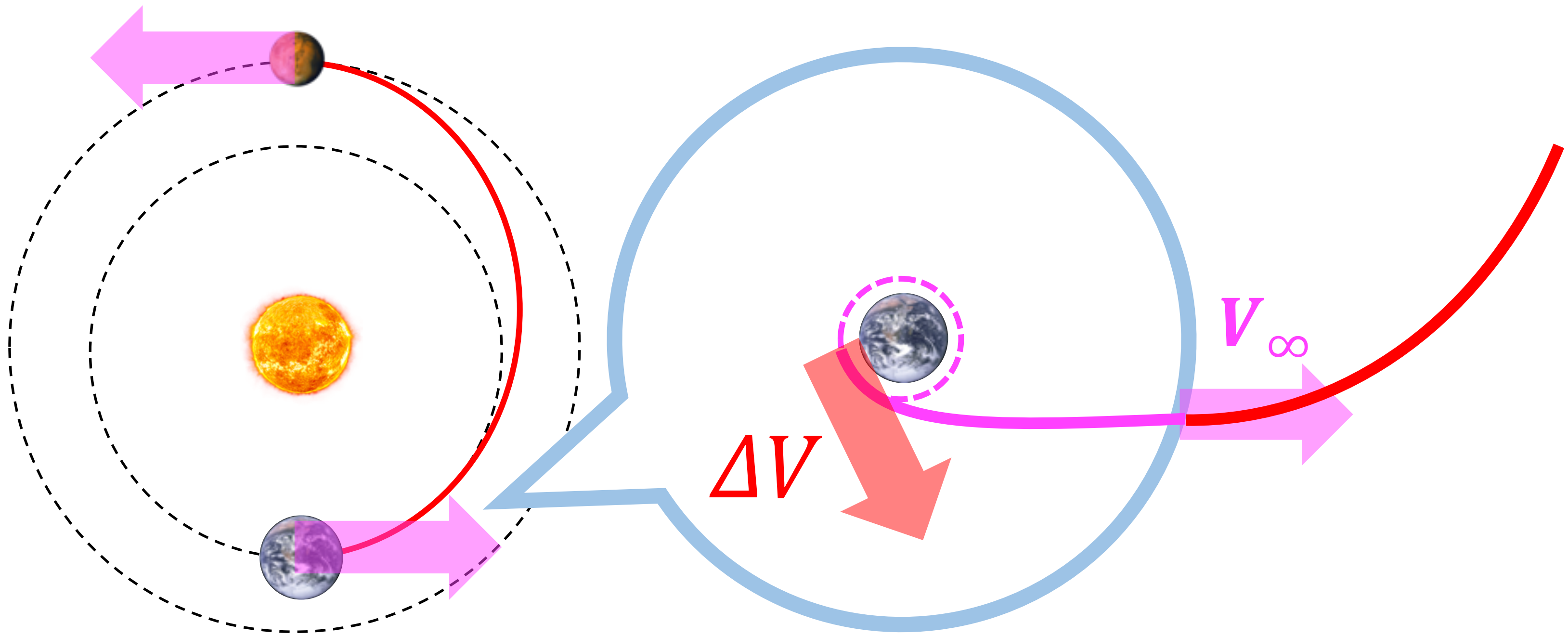
In Reality…
(Considering the Gravity of the Earth)



This approximated design approach is called

Patched Conics

In Reality…
(Considering the Gravity of the Earth)



What is the required ΔV to escape from LEO with the hyperbolic excess velocity V_∞?

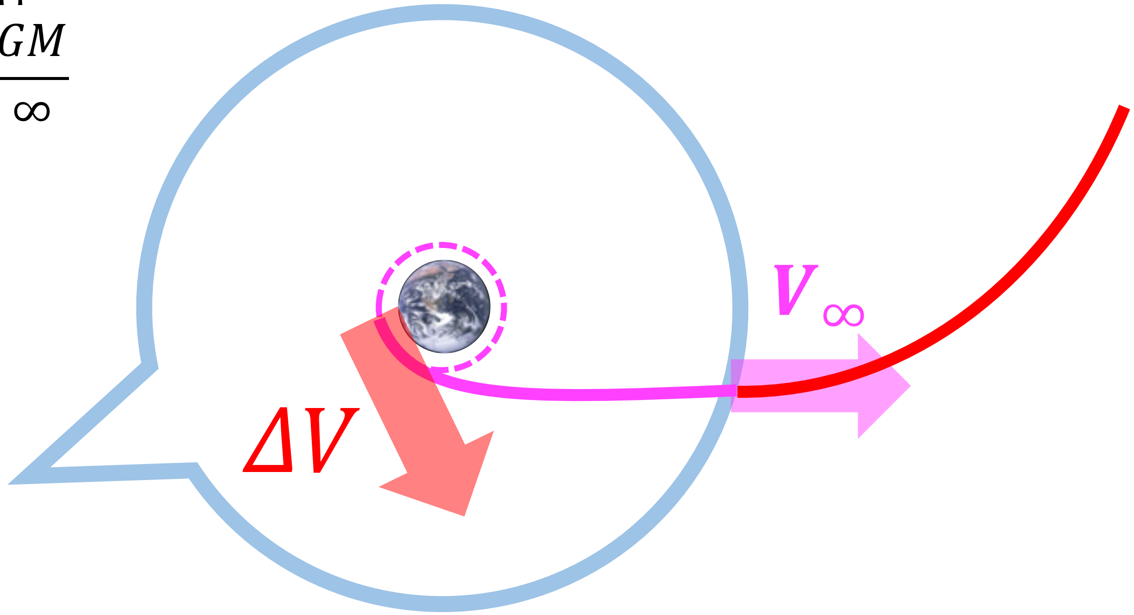
Earth Departure ΔV

From energy conservation

$$\frac{1}{2}v_0^2 - \frac{GM}{r_0} = \frac{1}{2}V_\infty^2 - \frac{GM}{\infty}$$

Hence

$$\frac{1}{2}v_0^2 - \frac{GM}{r_0} = \frac{1}{2}V_\infty^2$$



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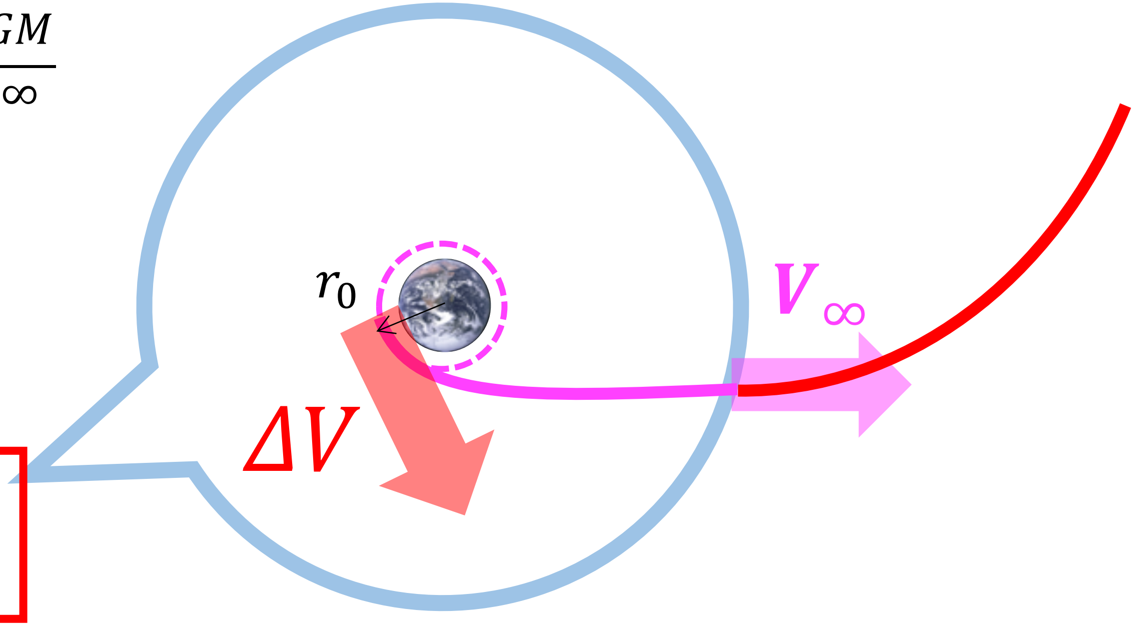
$$\frac{1}{2}v_0^2 - \frac{GM}{r_0} = \frac{1}{2}V_\infty^2$$

Solving for v_0 yields

$$v_0 = \sqrt{V_\infty^2 + \frac{2GM}{r_0}}$$



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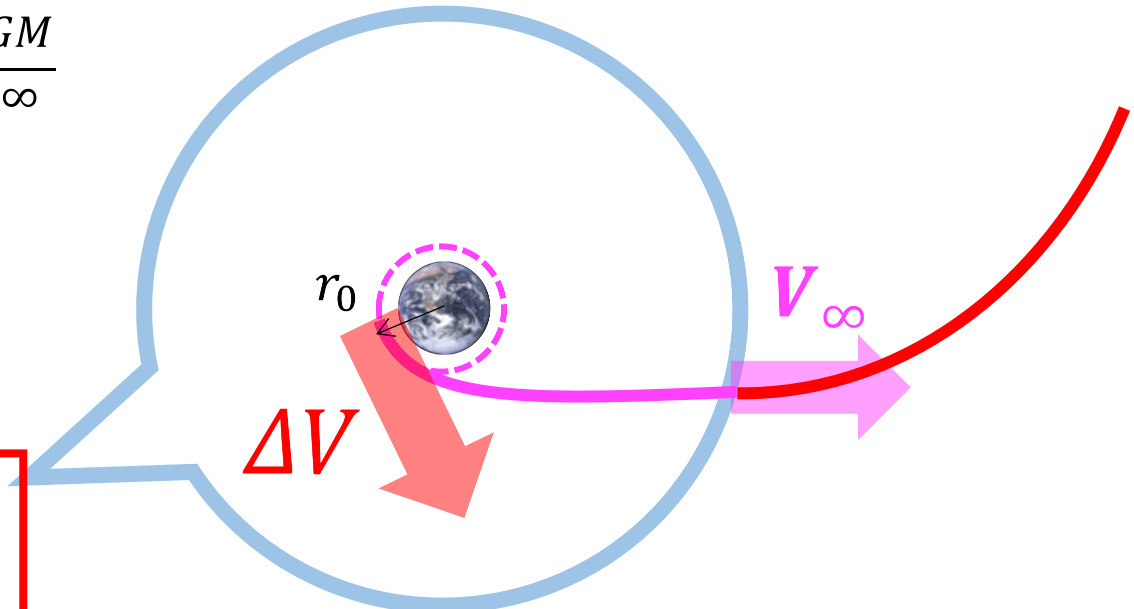

Subtracting the velocity of the initial circular orbit $\sqrt{\frac{GM_E}{r_0}}$, we obtain the departure ΔV as

$$\Delta V = \sqrt{V_\infty^2 + \frac{2GM_E}{r_0}} - \sqrt{\frac{GM_E}{r_0}}$$

Exercise: Carriable Dry Mass to Saturn

Problem 1: Using the Hohmann transfer orbit, calculate V_∞ to reach Saturn.

Condition：

- The gravity constant of the Sun $GM = 1.327 \times 10^{11}$ (km^3/s^2)
- The Earth moves in a circular orbit with $r_p = 1.496 \times 10^8$ (km)
- Saturn moves in a circular orbit with $r_a = 1.427 \times 10^9$ (km)

Problem 2：Calculate the ΔV required from LEO to Saturn.

Condition：

- The gravity constant of the Earth $GM_E = 3.986 \times 10^5$ (km^3/s^2)
- The spacecraft is initially in a circular orbit with $r_0 = 6.678 \times 10^3$ (km)

Problem 3：Calculate the carriable mass (dry mass) to Saturn.

Condition：

- Initial mass $m_0 = 1.5\mathrm{t}$
- Specific impulse of rocket $I_{sp} = 280s$

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- Satur…

Pr…

… (km^3/s^2)

- … orbit with $r_0 = 6.678 \times 10^3$ (km)

Probl… 3 : Calculate the carriable mass (dry mass) to Saturn.

Condition :

- Initial mass $m_0 = 1.5\text{t}$
- Specific impulse of rocket $I_{sp} = 280s$

The 1.5t rocket can carry only 105kg (including structural mass of the rocket)!!

Special Move of Astrodynamics!!

Special Technique to Gain Energy

clickest
Code: ayitqs

Question : What is the name of special technique for obtaining large energy in astrodynamics.

A

Slingshot

B

Swing-by

C

Gateway

D

Gum-Gum Bazooka

clickest
Code: ayitqs

Special Technique to Gain Energy

Question： What is the name of special technique for obtaining large energy in astrodynamics.

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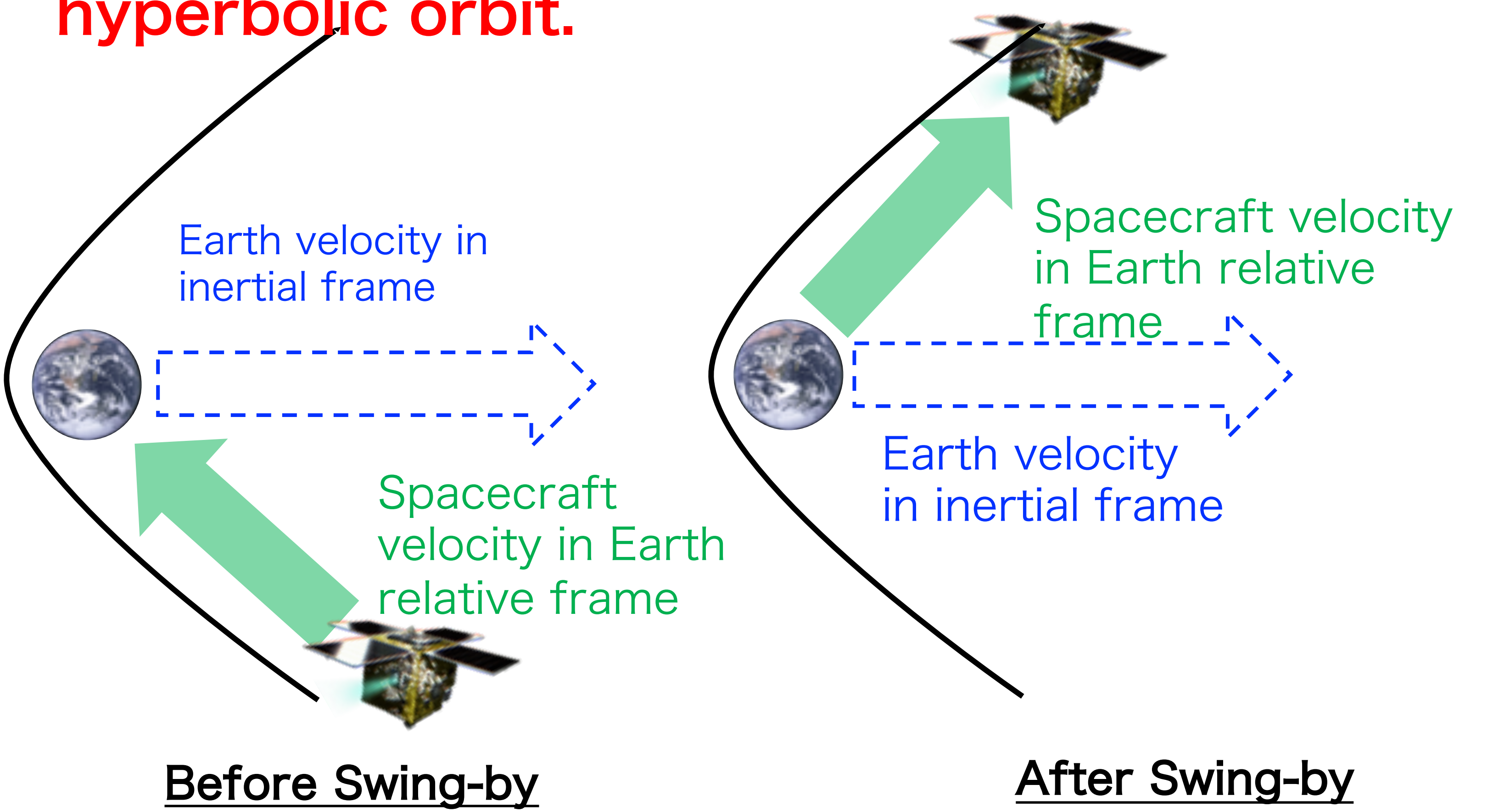
Swing-by, Gravity Assist, Slingshot

Swing-by is based on the same phenomena as collision
A key point to understand swing-by is “reference frame”

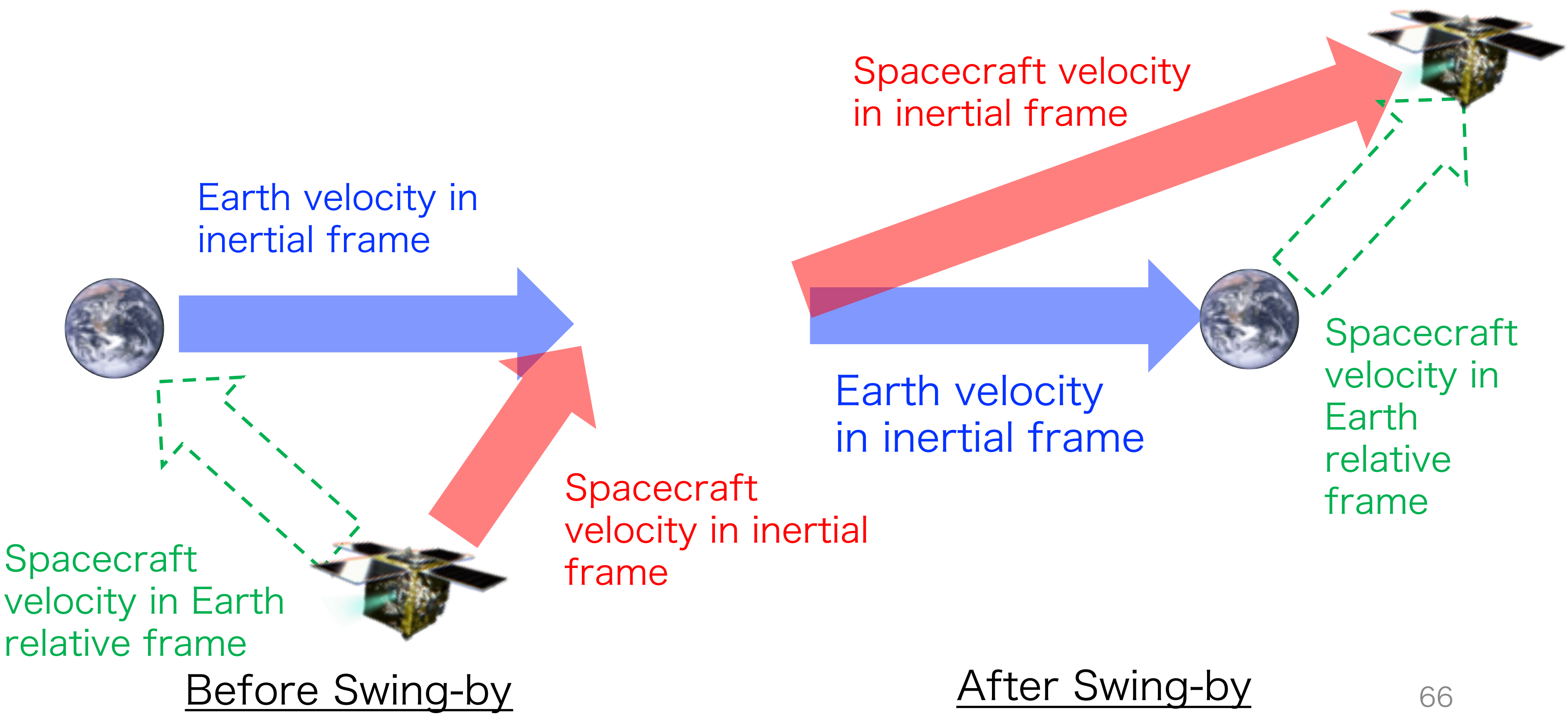
In Earth relative frame, the spacecraft orbit is a hyperbolic orbit.



Swing-by, Gravity Assist, Slingshot

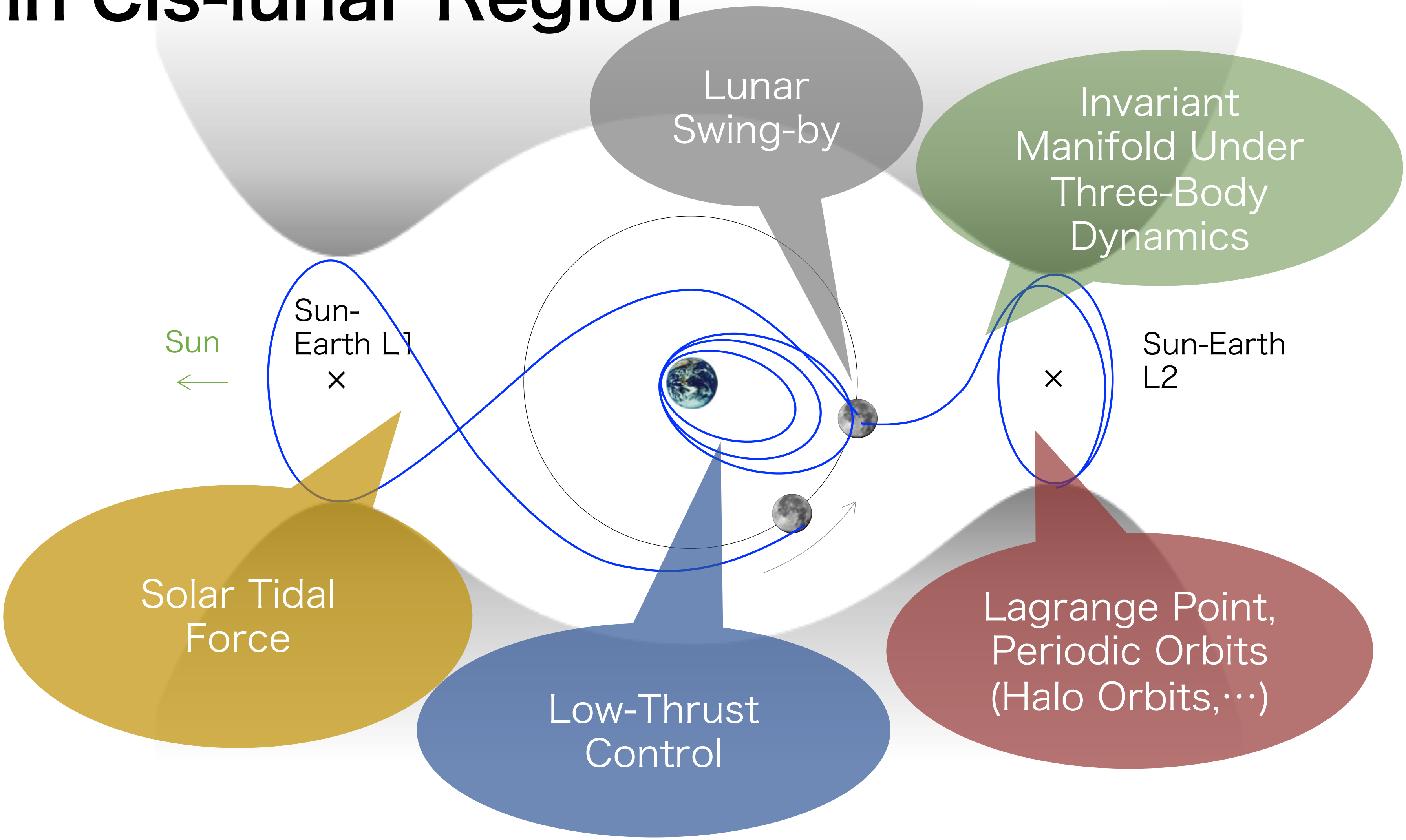
Swing-by is based on the same phenomena as collision
A key point to understand swing-by is “reference frame”

In inertial frame, the spacecraft is accelerated!!



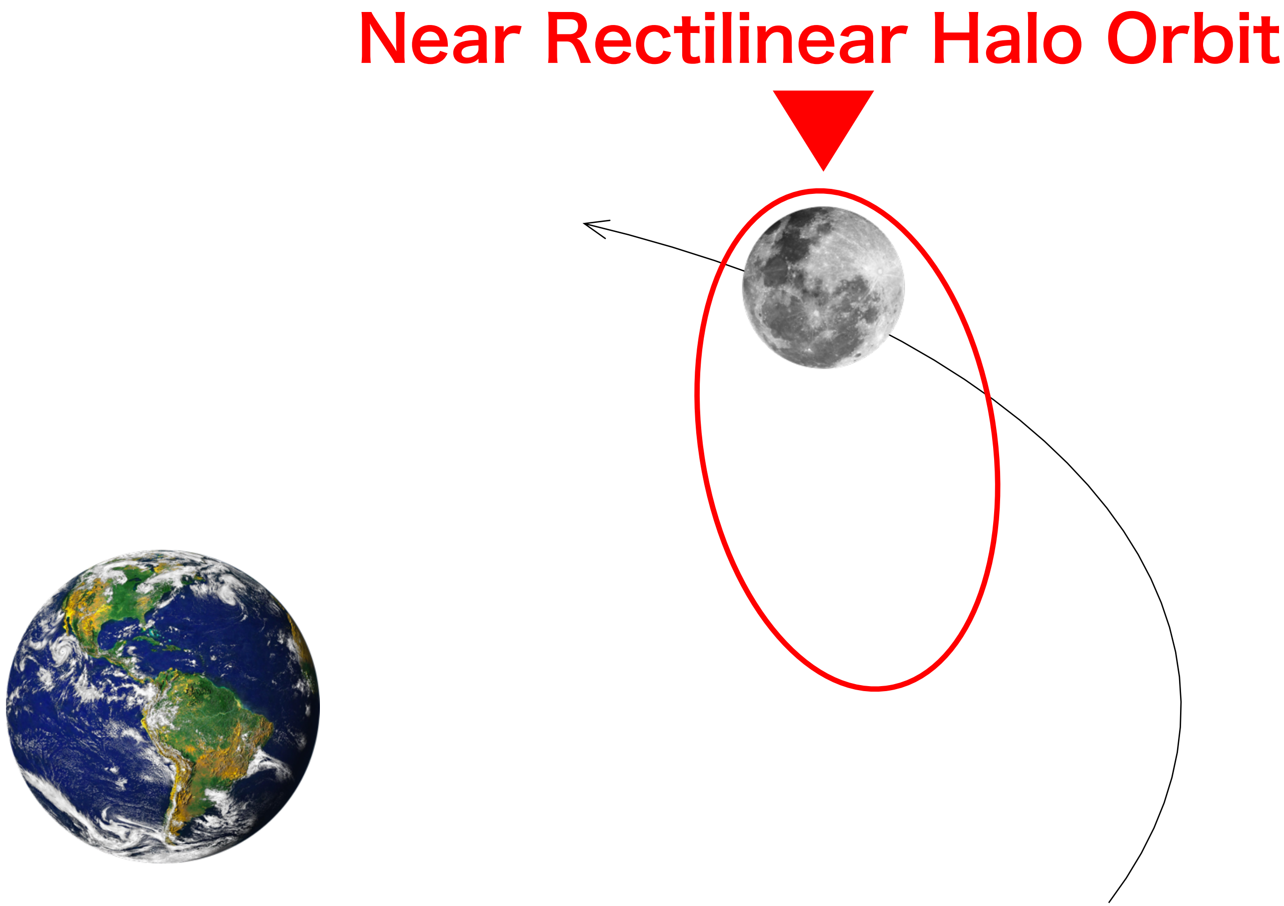
3. Advanced Techniques of Astrodynamics

Astrodynamics Techniques in Cis-lunar Region



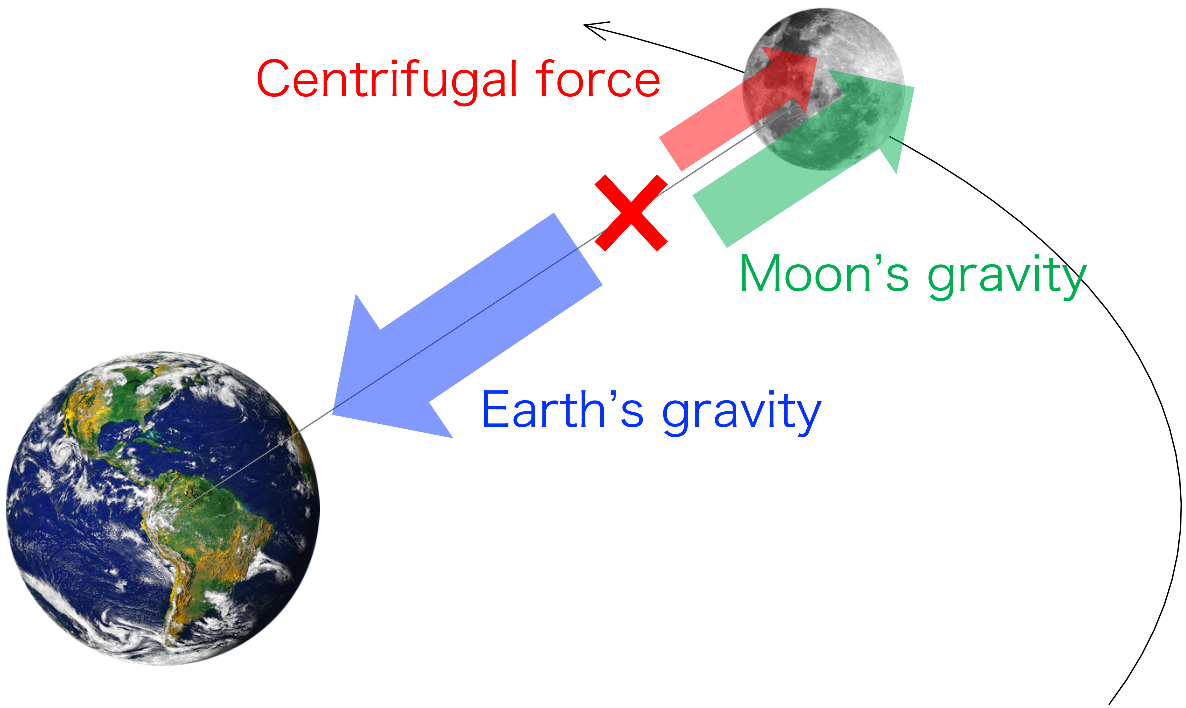
We can effectively design spacecraft trajectories using advanced astrodynamics techniques!!

Where is Lunar Orbital Platform-Gateway (LOP-G)??



A type of Halo orbits under the Earth and Moon gravity.

Where is Lunar Orbital Platform-Gateway (LOP-G)??

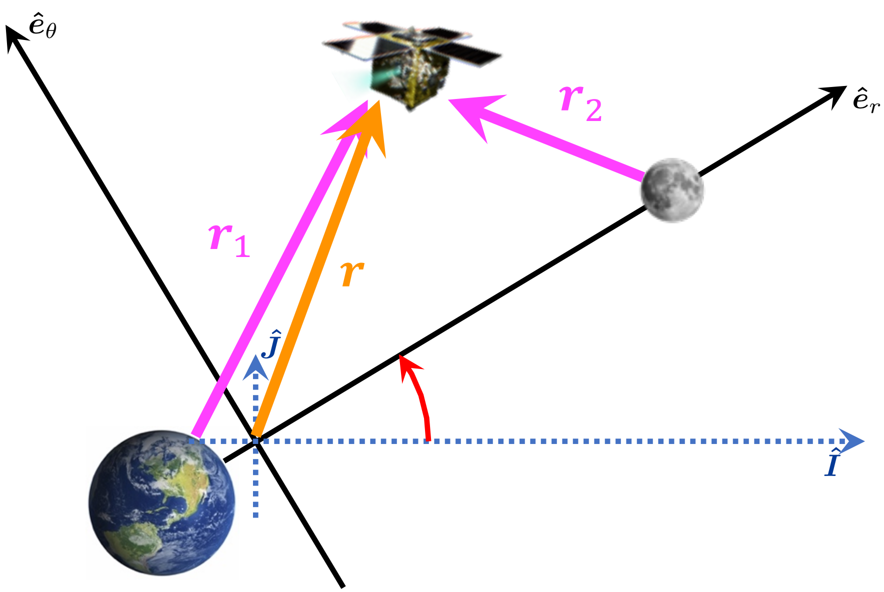


The equilibrium point where the earth's gravity, the moon's gravity, and the centrifugal force balance each other is called **the Lagrange point**.

Circular Restricted Three-Body Problem (CRTBP)

Equations of motion for three-body problems in inertial systems

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM_1}{r_1^3}\boldsymbol{r}_1 - \frac{GM_2}{r_2^3}\boldsymbol{r}_2$$



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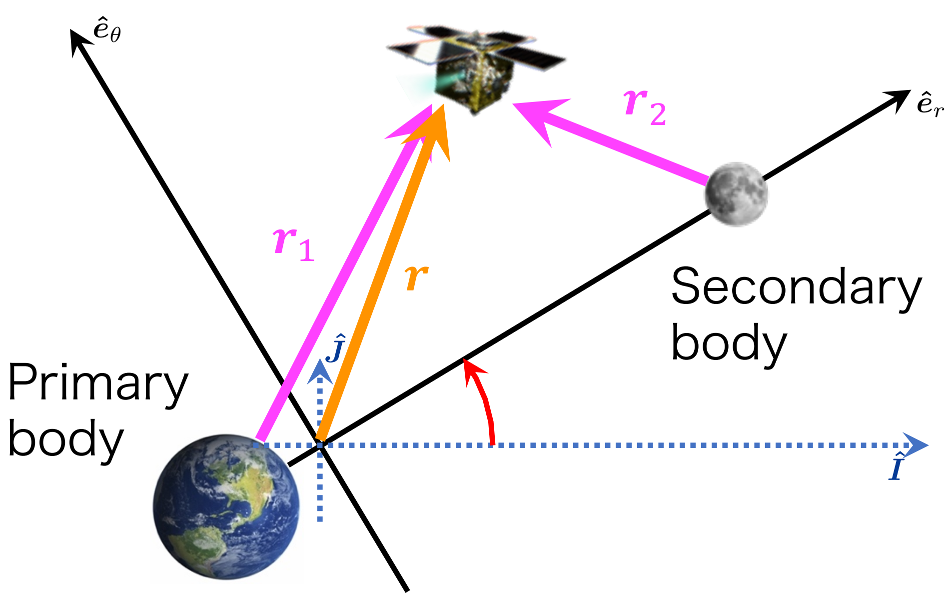


Assumption 1: Restricted
Assume that the gravity of the spacecraft (i.e. the third body) does not affect the other two objects.

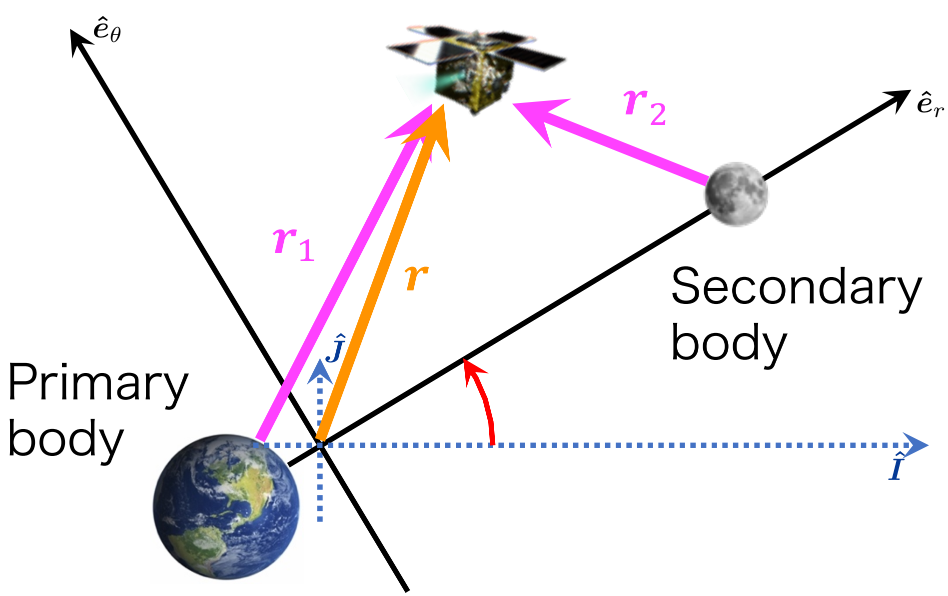
Assumption 2: Circular
Assume that both primary and secondary bodies move in a circular orbit around the barycenter.

Circular Restricted Three-Body Problem (CRTBP)

Equations of motion of CRTBP in a rotating coordinate system

$$\begin{cases} \ddot{x} - 2\,\dot{y} &= \Omega_x, \\ \ddot{y} + 2\,\dot{x} &= \Omega_y, \\ \ddot{z} &= \Omega_z, \end{cases} \qquad \Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\bar{\mu}}{r_1} + \frac{\bar{\mu}}{r_2}$$

where $\bar{\mu} = \dfrac{m_2}{m_1 + m_2}$ $\qquad \Omega_x = \dfrac{\partial \Omega}{\partial x} \quad \Omega_y = \dfrac{\partial \Omega}{\partial y} \quad \Omega_z = \dfrac{\partial \Omega}{\partial z}$



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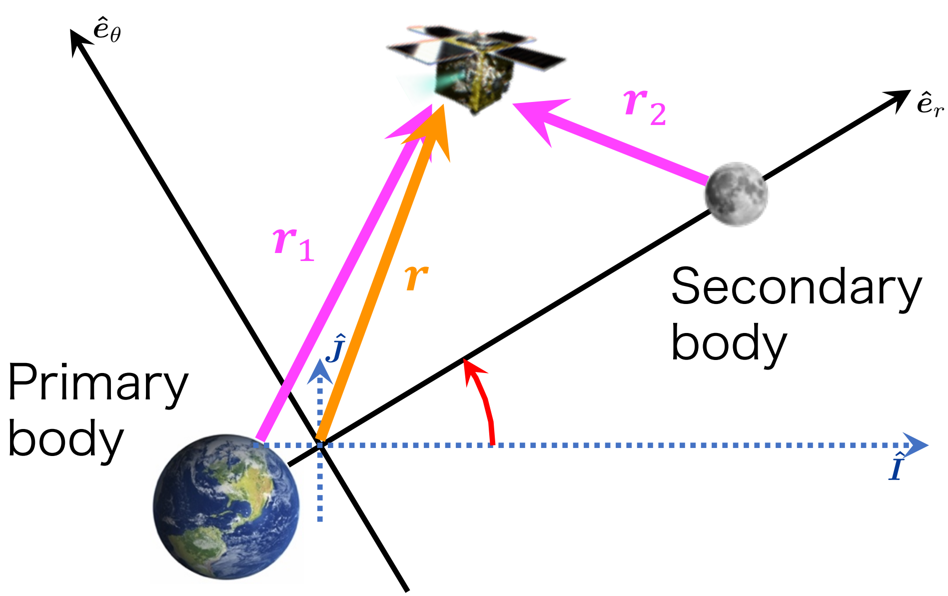
=0

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\bar{\mu}}{r_1} + \frac{\bar{\mu}}{r_2}$$

where $\bar{\mu} = \dfrac{m_2}{m_1 + m_2}$

$$\Omega_x = \frac{\partial \Omega}{\partial x} \quad \Omega_y = \frac{\partial \Omega}{\partial y} \quad \Omega_z = \frac{\partial \Omega}{\partial z}$$



The equilibrium point can be calculated using the dynamical systems theory technique.

(The equilibrium points = Lagrange points)

Number of Lagrange Points

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Code: ayitqs

Question: How many Lagrange points exist in CRTBP?

A

0

B

3

C

5

D

7

Number of Lagrange Points

clickest
Code: ayitqs

Question: How many Lagrange points exist in CRTBP?

A

0

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3

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5

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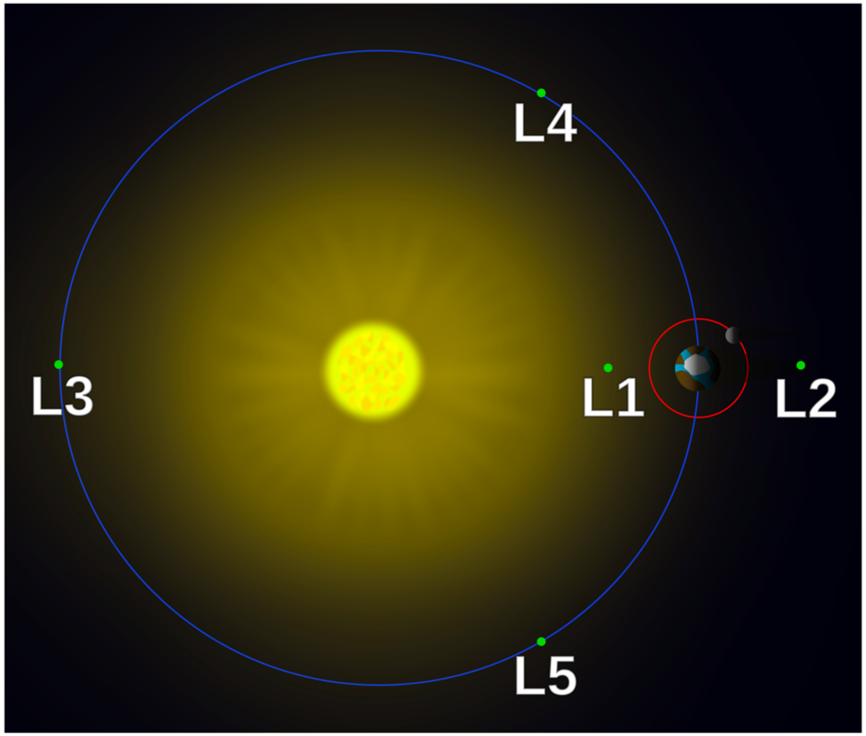
7

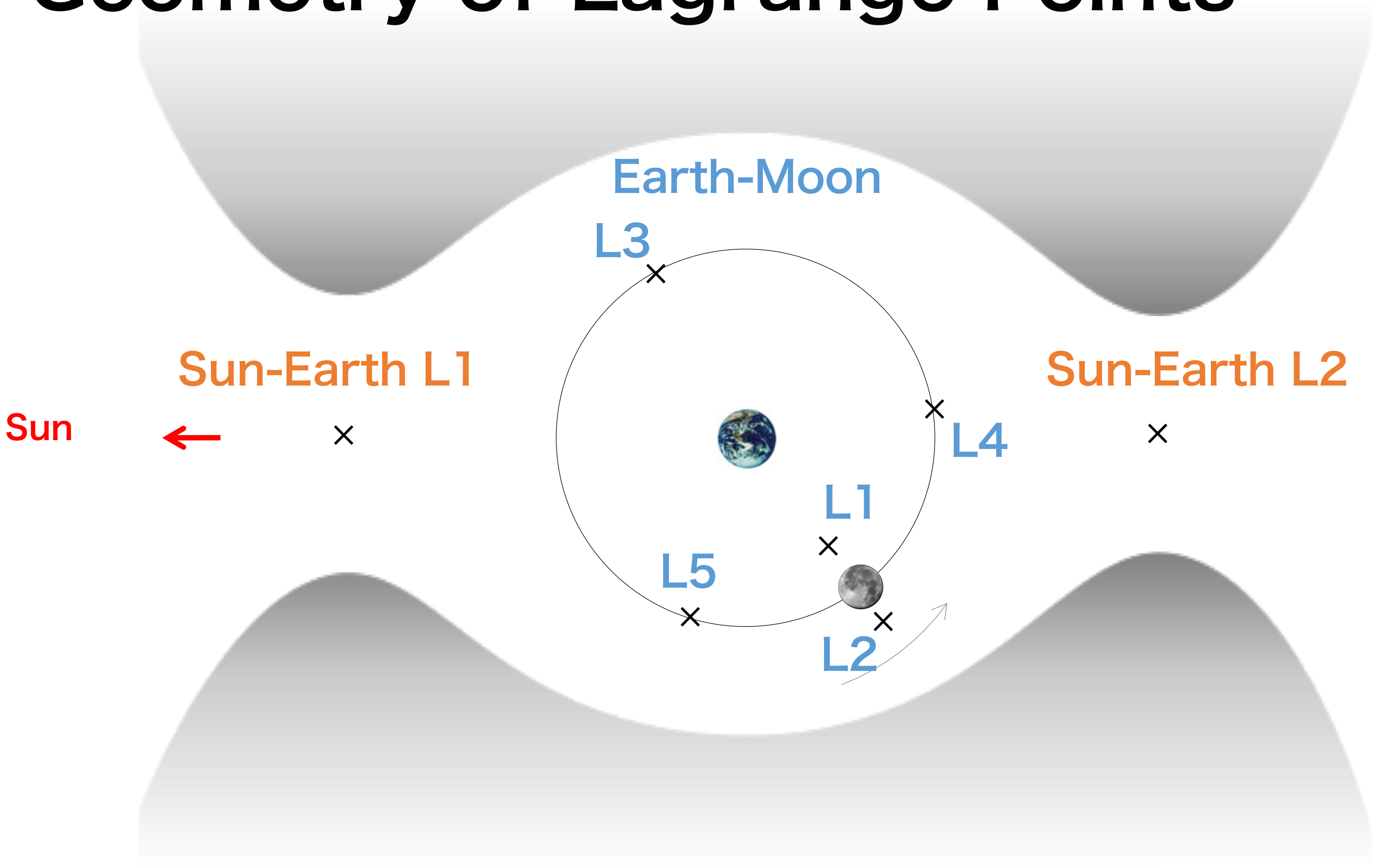
Lagrange Points



In general; five types of Lagrange points exist in any CRTBP system.

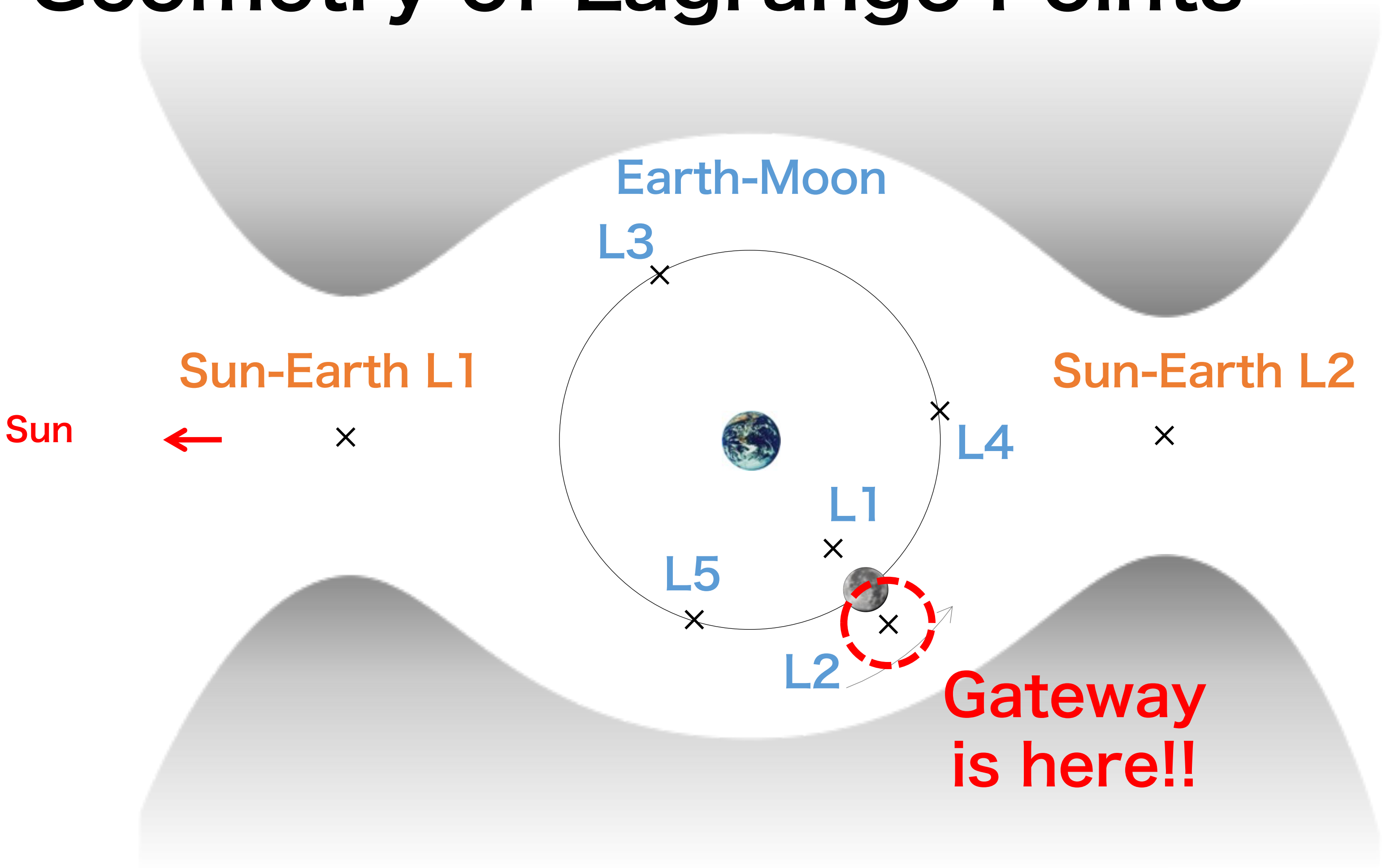
Ex) Earth-Moon L2 Lagrange point
Sun-Earth L1 Lagrange point

Geometry of Lagrange Points



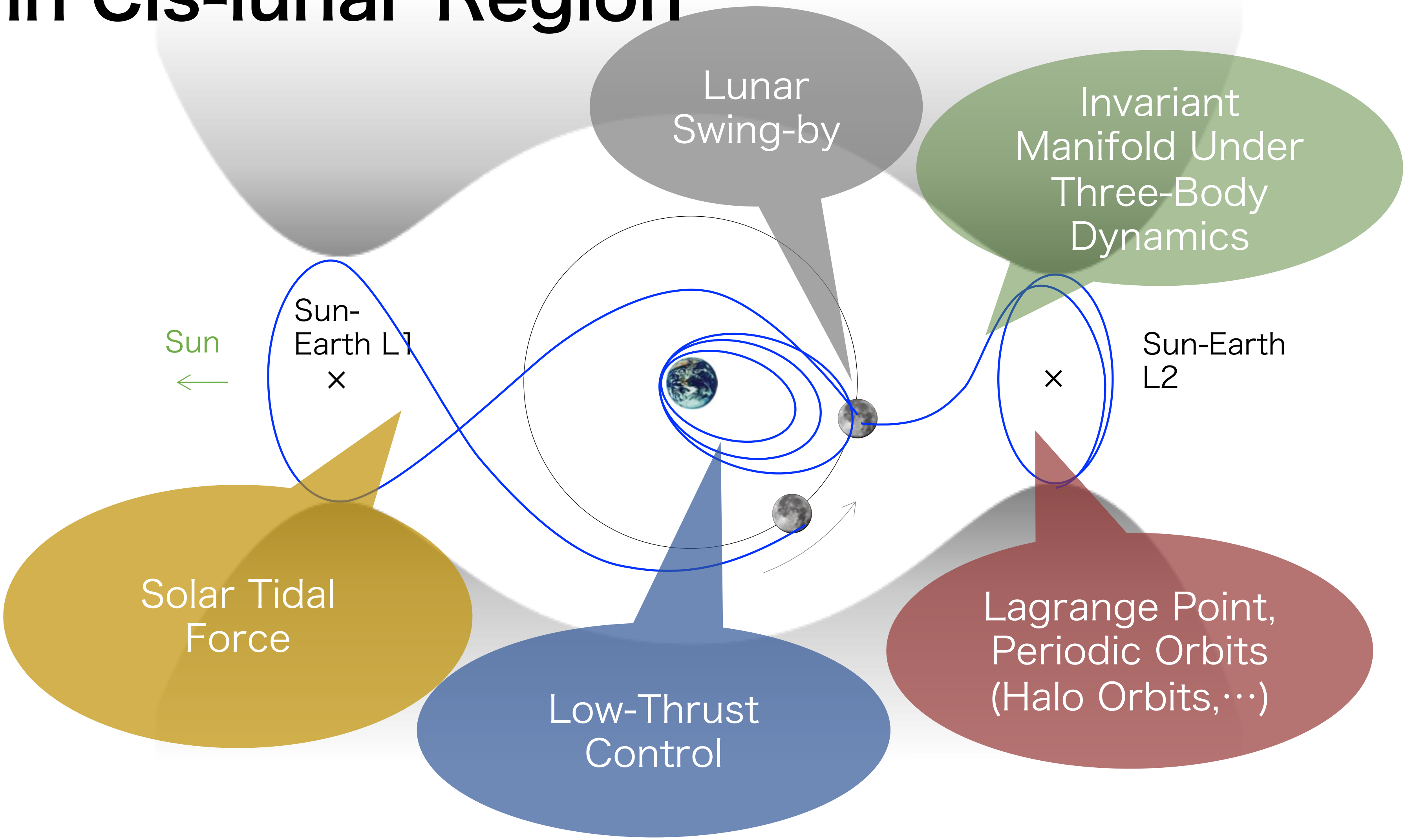
Sun-Earth line fixed rotational frame.

Geometry of Lagrange Points



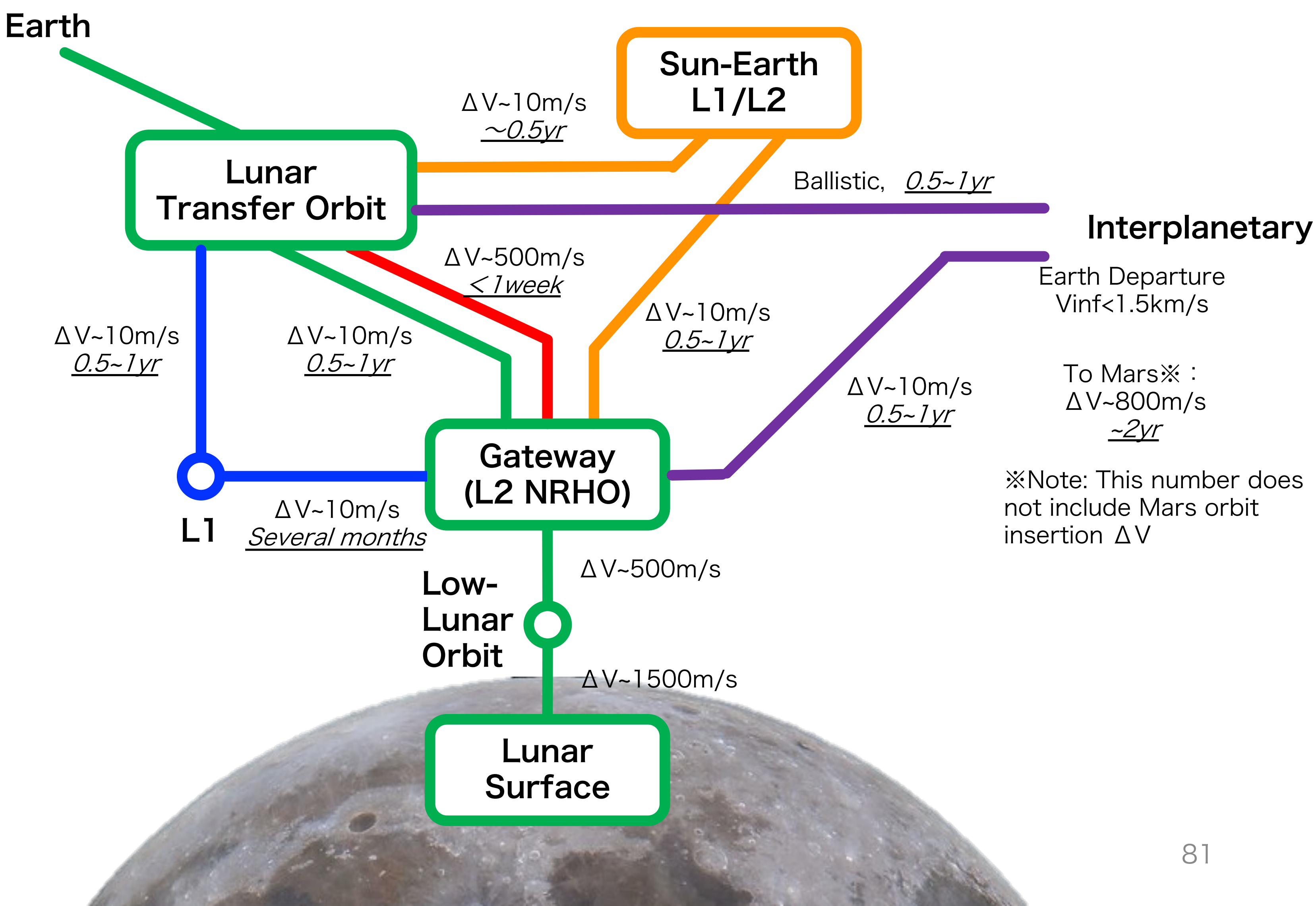
Sun-Earth line fixed rotational frame.

Astrodynamics Techniques in Cis-lunar Region



<u>**We can effectively design spacecraft trajectories using advanced astrodynamics techniques!!**</u>

Gateway ΔV Maps



4. Conclusions

Goal of This Lecture

- ☑ To be able to explain the role of trajectory design in deep space exploration missions.
- ☑ To be able to explain what the Hohmann transfer orbit, patched conics, and swing-by.
- ☑ To understand the brief overview of the advanced techniques of astrodynamics

For those who want to know more…

Books

- (Two-body Problem) Richard H. Battin, "An Introduction to the Mathematics and Methods of Astrodynamics", AIAA Education Series, 1999.
- (Trajectory Optimization) John T. Betts, "Practical Methods for Optimal Control Using Nonlinear Programming," SIAM Advances in Design and Control, 2001.
- (Three-Body Problem) Wang Sang Koon, Martin W. Lo, Jerrold E. Marsden, Shane D. Ross, "Dynamical Systems, the Three-Body Problem and Space Mission Design," 2011.

Keywords

- Two-body Problem: (Keplerian) Orbital Elements, Lambert's Problem, Swing-by, V-infinity Leveraging Transfer, Resonant Orbits
- Three-Body Problem: Stability, Periodic Orbits (Halo Orbits), Invariant Manifolds
- Trajectory Optimization: Low-Thrust Trajectory Optimization, Direct Method, Indirect Method, Nonlinear Programming

Tools

- NAIF SPICE Toolkit (https://naif.jpl.nasa.gov/naif/toolkit.html)
- NASA GMAT (https://software.nasa.gov/software/GSC-17177-1)
- Global Trajectory Optimization Tool: PyGMO (https://esa.github.io/pygmo2/)
- Global Trajectory Optimization Tool: EMTG(https://github.com/nasa/EMTG)

Exercise: Carriable Dry Mass to Saturn

Problem 1: Using the Hohmann transfer orbit, calculate V_∞ to reach Saturn.

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- Saturn moves in a circular orbit with $r_a = 1.427\times10^{9}$ (km)

Answer: Using the equation of the Hohmann transfer orbit

$$V_\infty = \sqrt{\frac{GM}{r_p}}\sqrt{\frac{2r_a}{r_a + r_p}} - \sqrt{\frac{GM}{r_p}} = 10.29\ \mathrm{km/s}$$

Exercise: Carriable Dry Mass to Saturn

Problem 2 : Calculate the ΔV required from LEO to Saturn.

Condition :

- The gravity constant of the Earth $GM_E = 3.986 \times 10^5$ (km³/s²)
- The spacecraft is initially in a circular orbit with $r_0 = 6.678 \times 10^3$ (km)

Answer: Under the patched-conics assumption,

$$\Delta V = \sqrt{V_\infty^2 + \frac{2GM_E}{r_0}} - \sqrt{\frac{GM_E}{r_0}} = 7.282\ \mathrm{km/s}$$

where $V_\infty = 10.29\ \mathrm{km/s}$.

Exercise: Carriable Dry Mass to Saturn

Problem 3：Calculate the carriable mass (dry mass) to Saturn.

Condition：

- Initial mass $m_0 = 1.5\text{t}$
- Specific impulse of rocket $I_{sp} = 280s$

Answer: Using Tsiolkovsky's rocket equation with $\Delta V = 7.282\ \text{km}/s$,

$$m_T = m_0 \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right) = 105.6\ \text{kg}$$